On the determination of the α parameter in a μSR experiment

T. Prokscha, PSI

March 30, 2007

Abstract

This article is just a collection of equations leading to Eq. (11) which allows to estimate α.

1 How to estimate α

Assumptions:

1. "forward" and "backward" detectors under $\phi = \pi$
2. forward and backward decay asymmetries A_F and A_B are the same, $A_F(t) = A_B(t) = A(t)$.
3. muon spin pionts initially to forward detector.

The forward and backward histograms $N_F(t)$ and $N_B(t)$ can be written as

$$\begin{aligned} N_F(t) &= N_F^0 \cdot \exp(-t/\tau_\mu) \cdot [1 + A(t)] + b_F \quad (1) \\ N_B(t) &= N_B^0 \cdot \exp(-t/\tau_\mu) \cdot [1 - A(t)] + b_B \quad (2) \\ \alpha N_F^0 &= N_B^0 \quad (3) \end{aligned}$$

where $N_{F,B}^0$ are the respective normalizations, τ_μ is the lifetime of the muon, and $b_{F,B}$ represent the flat background in each histogram. Rewrite Eqs. (1) and (2):

$$\begin{aligned} \frac{N_F(t) - b_F}{N_F^0} \cdot \exp(t/\tau_\mu) &= 1 + A(t) \quad (4) \\ \frac{N_B(t) - b_B}{N_B^0} \cdot \exp(t/\tau_\mu) &= 1 - A(t). \quad (5) \end{aligned}$$

Now, add Eqs. (4) and (5) to obtain

$$FB(t) \equiv \left(\frac{N_F(t) - b_F}{N_F^0} + \frac{N_B(t) - b_B}{N_B^0} \right) \cdot \exp(t/\tau_\mu) = 2. \quad (6)$$

Now, we have to determine the background and the normalization factors $N_{F,B}^0$. This can be done by adding Eqs. (1) and (2) to obtain the sum of the forward and backward histograms with $b = b_F + b_B$ the sum of background:

$$\begin{aligned} N_F(t) + N_B(t) &= N_F^0 \cdot \exp(-t/\tau_\mu) \cdot [1 + A(t)] + b_F \\ &+ N_B^0 \cdot \exp(-t/\tau_\mu) \cdot [1 - A(t)] + b_B \\ &= (N_F^0 + N_B^0) \cdot \exp(-t/\tau_\mu) + A(N_F^0 - N_B^0) \cdot \exp(-t/\tau_\mu) + b \\ &= \left[(N_F^0 + N_B^0) + A(N_F^0 - N_B^0)\right] \cdot \exp(-t/\tau_\mu) + b \\ &= (N_F^0 + N_B^0)(1 + A\frac{N_F^0 - N_B^0}{N_F^0 + N_B^0}) \cdot \exp(-t/\tau_\mu) + b \\ &= (N_F^0 + N_B^0)\left(1 + A\frac{1-\alpha}{1+\alpha}\right) \cdot \exp(-t/\tau_\mu) + b \\ &= N_F^0(1+\alpha)\left(1 + A\frac{1-\alpha}{1+\alpha}\right) \cdot \exp(-t/\tau_\mu) + b \quad (7) \\ &\equiv N^0 \cdot \exp(-t/\tau_\mu) + b. \quad (8) \end{aligned}$$

Fitting Eq. (8) to the sum histogram yields N^0 and b. The single histogram background can then be obtained using the single detector rates $R_{F,B}$:

$$b_F = b \cdot \frac{R_F}{R_F + R_B} \tag{9}$$

$$b_B = b \cdot \frac{R_B}{R_F + R_B}. \tag{10}$$

Insertion of Eqs. (7) and (8) in Eq. (6) yields the "master" equation for the determination of α:

$$FB(t) = \left[N_F(t) - b_F + \frac{1}{\alpha}(N_B(t) - b_B)\right] \frac{1+\alpha}{N^0} \left(1 + \overline{A}\frac{1-\alpha}{1+\alpha}\right) \exp(t/\tau_\mu) = 2, \tag{11}$$

where $\overline{A}$ is now the averaged asymmetry ($A = A(t)!$) and can be obtained from the background-corrected forward/backward histograms:

$$\overline{A} \equiv \frac{\alpha N_F - N_B}{\alpha N_F + N_B}, \tag{12}$$

where $N_{F,B}$ are the **total** number of muons in each histogram. For LEM top/bottom histograms or TF experiments with fields larger than a few hundred Gauss we can set $A = \overline{A} = 0$.

2 Procedure

Determine $FB(t)$ in Eq. 11 and fit a 0^{th}-order polynom $p0$ to $FB(t)$. Vary α to obtain $p0$ as close as possible to 2. Main problem: the procedure for Eq. 11 always gives a result very close to 2. With the typical LEM statistics of a total of 2-3 Mevents in Forward/Backward histograms a fit of Eq. 11 to the data has an error of 0.002 (i.e. the result of a fit may give 2.0005(20)). Therefore, the estimate of α is not straight forward and requires generally additional information, such as decay asymmetry at $t \le t_0$. On the other hand, the procedure can be used to check the data. For example: a real signal from the sample must always yield $FB(t)$ scattering around 2. In low-energy experiments ($E < 3keV$, LCCO, spin-glass) I found clear deviations in the first $100 - 200$ ns which is usually in the range of strange, fast relaxations. Using the prodecure of Eq. 11 therefore allows to check, at which time $t > t_0$ the signal is free from distortions due to backscattering etc., i.e. the time range where the signal comes from the sample only.

A root macro **getAlpha.C** exists which allows to run the procedure for α estimation. **However, it requires manual input and "expert" experience in LEM data analysis and is therefore not suited for general use at the moment!**

Procedures that worked for different conditions:

- ZF data (Ag sample plate, GaSb:Mn, LCCO):
  - include α in the calculation of $\overline{A}$ as in Eq. (12).
  - for a given α loop over bin sizes and fit $FB(t)$ for each bin widths.
  - depending on stats choose bin widths $\in [50, 99]$ or $[100, 199]$;
    it could be possible to change the fit interval as well:
    $[200, 9600]$ ns, or $[200, 11000]$ ns, $[200, 12300]$ ns.
  - Vary α until mean value $\overline{FB}$ of $FB(t) \sim 2.000000(5)$.
  - with this α check $FB(t)$ and $A(t)$ at $t \sim t_0$.
- TF B_{par}: does not work, because phase difference between L and R detector is less than 180°.
- TF B_{perp}, 100 GTF (ZnO, CdS):
  - Set $\overline{A} = 0$.
  - loop over bin widths $\in [100, 199]$ ns, fit FB.
  - Search α where $FB_{max} > 2 + \sigma_{\overline{FB}}$, where $\sigma_{\overline{FB}}$ is the square-root of the variance of the fitted $FB's$.