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Kapitza Resistance between Dielectrics and Metals in the Normal and Superconducting States*

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Change of the Kapitza resistance as a metal goes from the normal state to the superconducting state is estimated in the dielectric-metal junction by extending Little's theory that the interaction between surface waves and conduction electrons contributes to heat transfer in the liquid-solid junction. According to this estimate, no change is expected in the case that sound velocities of the dielectric are larger than those of the metal. The sapphire-indium junction is such a case. The Kapitza resistance of this junction measured by the authors from 0.6°K to 2.2°K shows, however, a small difference between the nornal and the superconducting states of indium contrary to the above estimation.

§ 1. Introduction

Since Kapitza[1)] discovered the thermal boundary resistance at an interface between He II and solids, many investigations of this "Kapitza resistance" have been reported.[2)] According to Khalatnikov,[3)] transfer of heat from a solid to helium takes place by the radiation of various phonons and the Kapitza resistance arises due to the acoustic mismatch of the two substances. Little[4)] extended Khalatnikov's theory to heat transfer at an interface between any two dissimilar, isotropic solids. In his paper, he pointed out possibility that conduction electrons could contribute to heat transfer through the interaction with surface disturbances caused by totally reflected sound waves. This interaction is important when the acoustic velocities are very different in the two media. This idea was applied by Little[5)] to the study of the Kapitza resistance at the interface between helium and a metal, where a large fraction of the phonons in the liquid would be totally reflected and the surface disturbances are created because the velocities of sound in liquid helium is much smaller than those in a metal. He expected a difference in the Kapitza resistance at the interface as a metal goes from the normal to the superconducting state, for the phonon-electron interaction necessary for this mechanism vanishes in the latter state. The difference was observed by Challis[6)] between liquid helium and lead in the normal and superconducting state. Little calculated heat transfer due to this mechanism assuming that only interaction between the longitudinal component of surface waves and conduction electrons is an important contribution. Challis and Cheeke[7)] pointed out later that this assumption corresponds to the condition $\omega\tau\gg1$ where ω is the main phonon frequency and τ is the relaxation time of conduction electrons. Also, Andreev[8)] calculated the effect of conduction electrons on the Kapitza resistance under the conditions $\omega\tau\ll1$ and furthermore $ql\gg1$ and $ql\ll1$ where q is the wave vector of the sound and l is the electron mean free path. When $ql\gg1$ he solved the kinetic equation of the elecron distribution function and showed that the electron-phonon interaction gives a contribution to the heat flow at the interface as large as that of phonon transmission within the critical angles. To the contrary, when $ql\ll1$, the conduction electrons barely contribute to the heat transfer.

Challis and Cheeke[7)] examined Little's and Andreev's theories with some numerical evaluations and found that the Andreev's calculation could be extended to the region $\omega\tau\gg1$ and that the agreement is essentially obtained with Little's calculation when interaction between conduction electrons and transverse waves is weak.

In this paper, we extended Little's theory to a dielectric-metal interface. We considered the contribution of conduction electrons and surface waves interaction to heat transfer and calculated it numerically for a certain range of the ratio of densities of the two media and the ratio of the velocities of longitudinal waves in two media. The details are presented in § 2.

Many experimental works have been carried

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out for the Kapitza resistance between liquid helium and a solid. The results vary for different surface conditions. Although it is technically difficult to produce the ideal dielectric-metal junction, at least we should be able to reduce the aging effect of the junction, which makes us easier to compare experimental results with theory. Neeper and Dillinger[9] measured the Kapitza resistance on the sapphire-indium junction in the temperature range from 1.1°K to 2.1°K and found conduction electrons contribute little. Very recently Wolfmeyer, Fox and Dillinger[10] made measurements on sapphire-indium and sapphire-lead junctions in the wider temperature range from 0.4 to 4.0°K. They found a pronounced difference of the Kapitza resistance between the normal and superconducting states below 1.5°K. We also made careful measurements of the Kapitza resistance of the sapphire-indium junction in order to check contribution of conduction electrons and surface waves interaction in the temperature region between 0.6°K and 2.2°K. It is important to widen the temperature region of measurements, because as temperature decreases, less number of quasi-particles are available, as Little pointed out.

We shall discuss what we expect from our estimation in § 2 with our experimental results and other group's results mentioned above.

§ 2. Theory

In order to solve the problem of the heat transfer by conduction electrons from a dielectric to a metal, we consider the propagation of the plane sound waves between these two solids.

Let us suppose (1) two semi-infinite isotropic solids contact at the plane $Z=0$, (2) sound waves can be considered to propagate independently and to be in the thermal equilibrium.

We devide our problem into two cases, (1) the longitudinal sound wave, (2) the transverse sound wave incident from the medium 1 (a dielectrics) ($Z>0$) to the medium 2 (a metal) ($Z<0$), respectively.

Case 1) Incidence of the longitudinal phonon.

The displacement vector of the incident sound wave may be determined by scalar potential φ_I, such that

$$u_I = \text{grad}\ \varphi_I\ . \qquad (1)$$

Let the incident wave vector k_{1l} lie in the XZ-plane as shown in Fig. 1. Futhermore let the angle of incidence of the longitudinal wave be θ_0, its velocity C_{1l} and its frequency ω.

We then have

$$\varphi_I = A_0 \exp\{ik_{1l}(X\sin\theta_0 - Z\cos\theta_0) - i\omega t\}\ , \qquad (2)$$

where

$$k_{1l} = \omega / C_{1l}\ .$$

In the medium 1, reflected sound waves are constituted of transverse wave and longitudinal wave. Their displacement vector may be written as

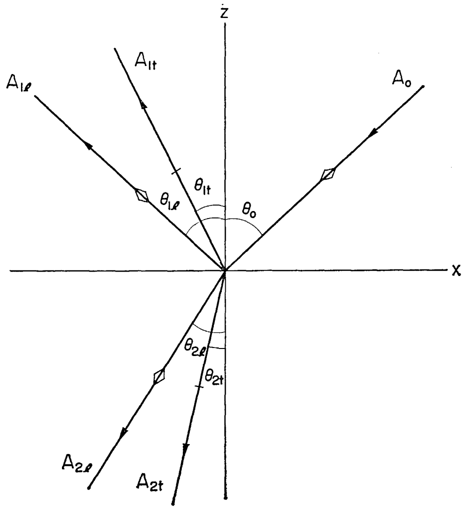

Fig. 1. The case that the incident wave is longitudinal.

$$u_R = \text{grad}\ \varphi_R + \text{rot}\ \Psi_R\ , \qquad (3)$$

where

$$\varphi_R = A_{1l} \exp\{ik_{1l}(X\sin\theta_{1l} + Z\cos\theta_{1l}) - i\omega t\}\ , \qquad (4)$$

$$(\Psi_R)_y = A_{1t} \exp\{ik_{1t}(X\sin\theta_{1t} + Z\cos\theta_{1t}) - i\omega t\}\ ,$$

$$(\Psi_R)_x = (\Psi_R)_z = 0\ . \qquad (5)$$

where $k_{1t} = \omega / C_{1t}$ and C_{1t} is the transverse sound velocity.

In the medium 2, the transmitted phonon may be also written as

$$u_T = \text{grad}\ \varphi_T + \text{rot}\ \Psi_T\ , \qquad (6)$$

where

$$\varphi_T = A_{2l} \exp\{ik_{2l}(X\sin\theta_{2l} - Z\cos\theta_{2l}) - i\omega t\}\ , \qquad (7)$$

$$(\Psi_T)_y = A_{2t} \exp\{ik_{2t}(X\sin\theta_{2t} - Z\cos\theta_{2t}) - i\omega t\}\ , \qquad (8)$$

$$(\Psi_T)_x = (\Psi_t)_z = 0\ .$$

here $k_{2l} = \omega / C_{2l}$, $k_{2t} = \omega / C_{2t}$, and C_{2l}, C_{2t} are the longitudinal and transverse sound velocities.

At the boundary plane $Z=0$, the following quantities must be continuous.

1) the normal components of the displacement

2) the tangential components of the displacement

3) the normal components of the stress
4) the tangential components of the stress

The stress tensor σ_{ik} is given by

$$\sigma_{ik}=2\rho C_t^2 U_{ik}+\rho(C_l^2-2C_t^2)U_{ll}\delta_{ik}\ , \qquad (9)$$

$$U_{ik}=\frac{1}{2}\left(\frac{\partial U_i}{\partial X_k}+\frac{\partial U_k}{\partial X_i}\right)\ . \qquad (10)$$

We then have the following relations as these boundary conditions.

$$\frac{\cos\theta_{1l}}{C_{1l}}A_{1l}+\frac{\sin\theta_{1t}}{C_{1t}}A_{1t}+\frac{\cos\theta_{2l}}{C_{2l}}A_{2l}-\frac{\sin\theta_{2t}}{C_{2t}}A_{2t}=\frac{\cos\theta_0}{C_{1l}}A_0\ , \qquad (11)$$

$$-\frac{\sin\theta_{1l}}{C_{1l}}A_{1l}+\frac{\cos\theta_{1t}}{C_{1t}}A_{1t}+\frac{\sin\theta_{2l}}{C_{2l}}A_{2l}+\frac{\cos\theta_{2t}}{C_{2t}}A_{2t}=\frac{\sin\theta_0}{C_{1l}}A_0\ , \qquad (12)$$

$$\rho_1\cos 2\theta_{1t}A_{1l}+\rho_1\sin 2\theta_{1t}A_{1t}-\rho_2\cos 2\theta_{2t}A_{2l}+\rho_2\sin 2\theta_{2t}A_{2t}=-\rho_1\cos 2\theta_{1t}A_0\ , \qquad (13)$$

$$\rho_1\left(\frac{C_{1t}}{C_{1l}}\right)^2\sin 2\theta_{1l}A_{1l}-\rho_1\cos 2\theta_{1t}A_{1t}+\rho_2\left(\frac{C_{2t}}{C_{2l}}\right)^2\sin 2\theta_{2l}A_{2l}+\rho_2\cos 2\theta_{2t}A_{2t}=\rho_1\left(\frac{C_{1t}}{C_{1l}}\right)^2\sin 2\theta_{1l}A_0\ . \qquad (14)$$

Also, we have one more relationship;

$$\frac{\sin\theta_0}{C_{1l}}=\frac{\sin\theta_{1l}}{C_{1l}}=\frac{\sin\theta_{1t}}{C_{1t}}=\frac{\sin\theta_{2l}}{C_{2l}}=\frac{\sin\theta_{2t}}{C_{2t}}\ . \qquad (15)$$

These equations from (11) to (14) may be solved easily with the above equation, and for instance, the amplitudes A_{2l}/A_0 and A_{2t}/A_0 are given by*

$$\frac{A_{2l}}{A_0}=\frac{1}{D}\cdot\frac{2\rho_1^2}{Z_{1l}}\left[2s^2\left(\rho_2-\rho_1\frac{C_{1t}^2}{C_{2t}^2}\right)\times\left(\frac{\rho_2}{Z_{2t}}-\frac{\rho_1}{Z_{1t}}\right)+\frac{\rho_1\rho_2}{Z_{1t}}+\frac{\rho_1\rho_2}{Z_{2t}}\right], \qquad (16)$$

$$\frac{A_{2t}}{A_0}=\frac{1}{D}\frac{2\rho_1^2}{Z_{1l}}\left[\frac{\rho_1\rho_2}{Z_{1t}Z_{2l}}\frac{s}{C_{2t}}(\rho_1C_{1t}^2-\rho_2C_{2t}^2)+\frac{2s}{C_{2t}}\left\{\rho_2-\rho_1+\frac{2s}{C_{2t}^2}\left(\rho_1C_{1t}^2-\rho_2C_{2t}^2\right)\right\}\right], \qquad (17)$$

where $s=\sin\theta_{2t}$ and D, Z_{1l}, Z_{1t}, Z_{2l}, and Z_{2t} are given by

$$D=\frac{\rho_1^2\rho_2^2}{Z_{1l}Z_{1t}Z_{2l}Z_{2t}}\frac{4s^2}{C_{2t}^2}(\rho_2C_{2t}^2-\rho_1C_{1t}^2)^2+\frac{\rho_1^2\rho_2^2}{Z_{2l}Z_{1t}}+\frac{\rho_1^2\rho_2^2}{Z_{1l}Z_{2t}}+\frac{\rho_1^2}{Z_{1l}Z_{1t}}\left\{\rho_2+\frac{2s^2}{C_{2t}^2}(\rho_1C_{1t}^2-\rho_2C_{2t}^2)\right\}^2$$
$$+\frac{s^2}{C_{2t}^2}\left\{\rho_1-\rho_2-\frac{2s^2}{C_{2t}^2}(\rho_1C_{1t}^2-\rho_2C_{2t}^2)\right\}^2+\frac{\rho_2^2}{Z_{2l}Z_{2t}}\left\{\rho_1+\frac{2s^2}{C_{2t}^2}(\rho_2C_{2t}^2-\rho_1C_{1t}^2)\right\}^2\ , \qquad (18)$$

$$Z_{1l}=\rho_1C_{1l}/\cos\theta_{1l}\ , \qquad (19)$$

$$Z_{1t}=\rho_1C_{1t}/\cos\theta_{1t}\ , \qquad (20)$$

$$Z_{2l}=\rho_2C_{2l}/\cos\theta_{2l}\ , \qquad (21)$$

$$Z_{2t}=\rho_2C_{2t}/\cos\theta_{2t}\ . \qquad (22)$$

When C_{1t} approaches to zero, only the longitudinal wave exists in medium 1, namely, medium 1 becomes liquid. The amplitude A_{2l}/A_0 given by the eq. (16) is reduced to

$$\frac{A_{2l}}{A_0}=\frac{\rho_1}{2\rho_2}\cdot\frac{2Z_{2l}\cos 2\theta_{2t}}{Z_{2l}\cos^2 2\theta_{2t}+Z_{2t}\sin^2 2\theta_{2t}+Z_{1l}}\ .$$

This result coincides with Khalatnikov's calculation for the Kapitza resistance of a liquid-solid interface.[11]

If $C_{2t}>C_{1l}$, when θ_0 becomes larger than $\sin^{-1}(C_{1l}/C_{2t})$, then,

$$\sin\theta_{2t}=C_{2t}\sin\theta_0/C_{1l}>1\ ,$$
$$\cos\theta_{2t}=i\sqrt{\sin^2\theta_{2t}-1}\ ,$$
$$(\Psi_T)_y=A_{2t}\exp\{ik_{2t}X\sin\theta_{2t}+k_{2t}Z\sqrt{\sin^2\theta_{2t}-1}-i\omega t\}\ , \qquad (23)$$

accordingly, $(\Psi_T)_y$ attenuates in the direction $Z<0$, and the amplitudes A_{1l}, A_{1t}, A_{2l} and A_{2t} become complex numbers. This means that the phases of A_{1l}, A_{1t}, A_{2l}, and A_{2t} deviate from zero. In this region of the incident angle the amplitudes A_{2l}/A_0 and A_{2t}/A_0 may have poles in complex s-plane. Generally speaking, it is hard to find where these poles are. In a special case that $\rho_1/\rho_2\ll 1$, they have a pole at $s=s_0+i\rho_1C_{1l}/(\rho_2C_{2l})$ as the case of liquid-solid junction, where s_0 is the solution greater than 1 in the following equation

$$16\left(1-\frac{C_{2t}^2}{C_{2l}^2}\right)s^6-\left(24-16\frac{C_{2t}^2}{C_{2l}^2}\right)s^4+8s^2-1=0\ . \qquad (24)$$

This pole, which corresponds to Rayleigh surface waves of a free body, plays an important role to the Kapitza resistance through the interaction between surface waves and free electrons in the liquid-solid junction. This interaction is the cause of change of the Kapitza resistance as the superconducting metal is going to the normal state. A contribution of Rayleigh surface waves to Kapitza resistance was formulated by Little[5] and Andreev[8] in the liquid-metal junction. Both theories coincide in magnitude of the Kapitza resistance by dropping the interaction between

* When θ_0 is less than $\sin^{-1}(C_{1l}/C_{2l})$, the energy ratio of the transmitted longitudinal and transverse sound waves to the incident wave are given by

$$\left(\frac{A_{2l,t}}{A_0}\right)^2\frac{\rho_2C_{1l}\cos\theta_{2l,t}}{\rho_1C_{2l,t}\cos\theta_0}\ .$$

conduction electrons and the transverse surface waves in Andreev's theory, which is appropriate for $T \gtrsim 0.01°K$ because of lower attenuation of transverse wave.[7] Then, when we extend this Little's theory to the dielectric-metal junction, we also neglect the interaction between the transverse surface waves and free electrons and consider only the interaction between the longitudinal surface waves and free electrons in a metal.

When θ_0 becomes as $\sin\theta_0 > C_{1l}/C_{2l}$, the longitudinal displacement vector $\boldsymbol{u}_l$ is expressed in a medium 2 as

$$\boldsymbol{u}_l = \mathrm{grad}\,\varphi_l ,$$
$$\varphi_l = \mathrm{Re}\sum_k A_{2l}\exp\{ik_{2l}X\sin\theta_{2l} + \eta_l Z - i\omega t\}$$
$$= \sum_k f_l(\theta)\{b_k \exp(ik_{2l}X\sin\theta_{2l} - i\omega t) + c.c.\}$$
$$\exp(\eta_l Z) , \tag{25}$$

where

$$\eta_l = k_{2l}\sqrt{\sin^2\theta_{2l} - 1} ,$$
$$f_l(\theta) = |A_{2l}/(2A_0)| ,$$
$$b_k = A_0 \exp\{i \arg(A_{2l}/A_0)\} .$$

The equation (25) corresponds to the eq. (1.15) of the reference 5). In order to calculate the heat transfer due to conduction electronsurface wave interaction $\dot{Q}_e$ at the boundary between a dielectric and a metal, one can use the eq. (25) according to Little's theory. Performing calculations as he did, we can obtain the heat transfer $\dot{Q}_e$;

$$\frac{\dot{Q}_e}{S} = \frac{8(eV_1)^2 m^2 k^4}{(2\pi)^5 \rho_2 \hbar^6 C_{2t}^3} J_4\left(\frac{\Theta}{T}\right) \cdot F_l T^3 \Delta T + \frac{(eV_1)^2 m k^6}{(2\pi)^5 \rho_2 E_f \hbar^6 C_{2t}^5} J_6\left(\frac{\Theta}{T}\right) \cdot G_l T^5 \Delta T , \tag{26}$$

where $\alpha = C_{1l}/C_{2l}$, Θ is the Debye temperature of a metal, eV_1 is the coupling constant, m is the mass of an electron, E_f, is Fermi energy and

$$J_n(x) = \int_0^x \frac{nx^{n-1}}{e^x - 1} dx , \tag{27}$$

$$F_l = \frac{\rho_2 C_{2t}^3}{\rho_1 C_{1l}^3} \int_{\sin^{-1}(C_{1l}/C_{2l})}^{\pi/2} \times \frac{\alpha^4 f_l^2(\theta)\sin\theta \tan^{-1}\{\sqrt{\sin^2\theta - \alpha^2}/\sin\theta\}}{\sin^2\theta - \alpha^2} d\theta, \tag{28}$$

$$G_l = \frac{\rho_2 C_{2t}^5}{\rho_1 C_{1l}^5} \int_{\sin^{-1}(C_{1l}/C_{2l})}^{\pi/2} \alpha^4 f_l^2(\theta)(\sin^2\theta - \alpha^2) d\theta . \tag{29}$$

Inserting reasonable numerical values into (26), obtains approximately

$$\dot{Q}_e = 7\times 10^4 F_l T^3 \Delta T + 2\times 10 G_l T^5 \Delta T (\mathrm{erg\cdot sec^{-1}\,cm^{-2}}) .$$

A numerical evaluations of F_l and G_l have been made for both solids with Poisson's ratio$=1/3$ (a typical value in many solids) and $\rho_2/\rho_1 = 1.0$ and $C_{2l}/C_{1l} = 3.0$. This gives

$$\dot{Q}_e = (4\times 10^3 T^3 + 5\times 10^{-1} T^5)\Delta T\,\mathrm{erg\cdot sec^{-1}\,cm^{-2}} ,$$

so the second term is negligible in the equation (26).

Case 2) Incidence of the transverse phonon.

When the transverse phonon is incident, we solve the problem in the same way as Case 1).

From the boundary conditions, we obtain the following equations

$$\frac{\cos\theta_{1l}}{C_{1l}} A_{1l} + \frac{\sin\theta_{1t}}{C_{1t}} A_{1t} + \frac{\cos\theta_{2l}}{C_{2l}} A_{2l} - \frac{\sin\theta_{2t}}{C_{2t}} A_{2t} = -\frac{\sin\theta_{1t}}{C_{1t}} A_0 , \tag{30}$$

$$-\frac{\sin\theta_{1l}}{C_{1l}} A_{1l} + \frac{\cos\theta_{1t}}{C_{1t}} A_{1t} + \frac{\sin\theta_{2l}}{C_{2l}} A_{2l} + \frac{\cos\theta_{2t}}{C_{2t}} A_{2t} = \frac{\cos\theta_{1t}}{C_{1t}} A_0 , \tag{31}$$

$$\rho_1\cos 2\theta_{1t} A_{1l} + \rho_1 \sin 2\theta_{1t} A_{1t} - \rho_2 \cos 2\theta_{2t} A_{2l} + \rho_2 \sin 2\theta_{2t} A_{2t} = \rho_1 \sin 2\theta_{1t} A_0 , \tag{32}$$

$$\rho_1\left(\frac{C_{1t}}{C_{1l}}\right)^2 \sin 2\theta_{1l} A_{1l} - \rho_1 \cos 2\theta_{1t} A_{1t} - \rho_2\left(\frac{C_{2t}}{C_{2l}}\right)^2 \times \sin 2\theta_{2l} A_{2l} - \rho_2 \cos 2\theta_{2t} A_{2t} = \rho_1 \cos 2\theta_{1t} A_0 . \tag{33}$$

These equations are similar to eqs. (11), (12), (13) and (14) and only the coefficients of A_0 are different from them. The amplitudes A_{2l}/A_0 and A_{2t}/A_0 are given by

$$\frac{A_{2l}}{A_0} = \frac{1}{D}\left[\frac{\rho_1^3\rho_2}{Z_{1l}Z_{1t}Z_{2t}} \cdot \frac{4s}{C_{2t}}(\rho_2 C_{2t}^2 - \rho_1 C_{1t}^2) + \frac{\rho_1^2}{Z_{1t}} \cdot \frac{2s}{C_{2t}}\left\{\rho_1 - \rho_2 + \frac{2s^2}{C_{2t}^2}(\rho_2 C_{2t}^2 - \rho_1 C_{1t}^2)\right\}\right],$$

$$\frac{A_{2t}}{A_0} = \frac{1}{D}\left[\frac{2\rho_1^2\rho_2}{Z_{1t}Z_{2l}}\left\{\rho_1 + \frac{2s^2}{C_{2t}^2}(\rho_2 C_{2t}^2 - \rho_1 C_{1t}^2)\right\} + \frac{2\rho_1^3}{Z_{1l}Z_{1t}}\left\{\rho_2 + \frac{2s^2}{C_{1t}^2}(\rho_1 C_{1t}^2 - \rho_2 C_{2t}^2)\right\}\right],$$

where D is given by the eq. (18). These amplitudes A_{2l}/A_0 and A_{2t}/A_0 have a pole each as the case of incidence of the longitudinal sound wave.

The heat transfer $\dot{Q}_e$ by the surface mode in a metal is experessed in the same equation (26) as Case 1) replacing F_l and G_l with F_t and G_t

$$F_t = \frac{2\rho_2 C_{2t}^3}{\rho_1 C_{1t}^3} \int_{\sin^{-1}(C_{1t}/C_{2l})}^{\pi/2} \times \frac{\beta^4 f_t^2(\theta)\sin\theta\tan^{-1}\{\sqrt{\sin^2\theta - \beta^2}/\sin\theta\}}{\sin^2\theta - \beta^2} d\theta ,$$

$$G_t = \frac{2\rho_2 C_{2t}^5}{\rho_1 C_{1t}^5} \int_{\sin^{-1}(C_{1t}/C_{2l})}^{\pi/2} \beta^4 f_t^2(\theta)(\sin^2\theta - \beta^2) d\theta ,$$

where

$$\beta = C_{1t}/C_{2l} ,$$

and also multiplying a factor 2 because there are two modes of the incident transverse waves. $f_t(\theta)=|A_{2l}/(2A_0)|$ should be calculated from eq. (34).

A numerical evaluation of eq. (26) for transverse waves have been made as the previous case. This gives $\dot{Q}_e=(1.3\times10^4T^3+7T^5)\Delta T$ erg $\sec^{-1}$ cm^{-2}, and therefore the second term is also negligible.

Adding the heat transfer of the both cases *i. e.* incidences of the longitudinal and transverse sound waves, we obtain $\dot{Q}_e$ as

$$\frac{\dot{Q}_e}{S}=\frac{8(eV_1)^2m^2k^4}{(2\pi)^5\rho_2\hbar^6C_{2t}^3}J_4\left(\frac{\Theta}{T}\right)FT^3\Delta T$$

$$=\frac{64\pi^5k^4\rho_0^4}{15h^6\rho_2C_{2t}^3}\lambda_2^2FT^3\cdot\Delta T , \qquad (36)$$

$$F=F_l+F_t ,$$

where we have put $eV_1=\lambda_2E_f$, $(\lambda_2\sim1)$ and have replaced the transport integral $J_4(\Theta/T)$ by its low temperature value of $4\pi^4/15$.

We calculated F numerically in Fig. 2 with Poisson's ratio 1/3 in both solids in a range of relative densities of the two media and the ratio of the velocities of longitudinal waves in the two media. From Fig. 2, a numerical evaluation of $\dot{Q}_e$ gives $\dot{Q}_e=1.7\times10^4T^3\Delta T$ erg $\sec^{-1}$ cm^{-2}, for $\rho_2/\rho_1=1.0$ and $C_{2l}/C_{1l}=3.0$. This value is comparable with the value $\dot{Q}_e=5.4\times10^3T^3\Delta T$ erg $\sec^{-1}$ cm^{-2} for the case of the interface between liquid helium and a metal,[5] and it is about 1/30 of phonon heat transfer at the interface between a solid and a solid.

When $C_{2t}/C_{1l}<1$, heat transfer $\dot{Q}_e$ is zero, because interaction of the surface modes with conduction electrons does not take place. The sapphire-indium junction for which we performed experiments is one of the examples of this case.

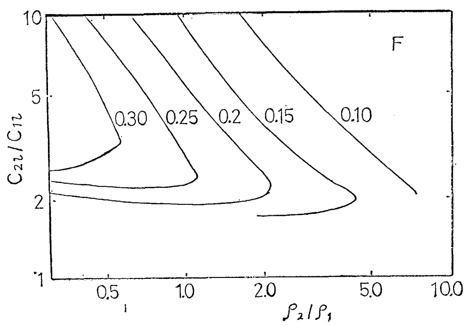

Fig. 2. The integral F with respect to the ratio of the velocity of the longitudinal waves in the two media and the relative densities. Poisson's ratio is equal to 1/3 in both media.

§ 3. Experiment

As mentioned in the introduction, conditions of the junction where the Kapitza resistance occurs are important factors in the comparison with theories. We should like to have a strain free, perfectly connected, strongly and uniformly bound, and smooth interface of very pure substances. It is not easy to find two substances which fill these requirements. We chose indium and sapphire and tried to fulfill them.

The samples were made by the same method as by Neeper's.[9] Both ends of a cylindrically shaped single crystal of sapphire, 30 mm long ×6 mm diameter were polished down to roughness of about 500Å. High purity indium (99.999%) was soldered on one face of sapphire by an ultrasonic soldering tool. The sapphire was then put in the tightly fit teflon mold, which is more than twice longer than the sapphire rod. This was mounted vertically in an induction furnace. Pieces of indium was placed directly above the mold and melted with an induction heater and casted in the form of a rod, 40 mm long×6 mm diameter on the indium soldered end of the sapphire. Then a narrow zone of the indium section was molten with an induction heater. This zone melting was done so slowly that when the metal solidifies, no conical hole due to shrinkage of solidification was left on the top of the indium. This procedure was repeated a few times. Indium and sapphire are so well bound that for an example, if we apply force to the rod perpendicular to its axis, the indium rod may be bent without any change at the interface. Then, the teflon mold was cut away and a heater was attached with varnish, GE 7031, on other end face of the sapphire as shown in the Figure 3. Finally, two copper wires with a carbon resistor thermometer for each were soldered around the indium rod near the boundaries and also two copper wires with thermometers were varnished around the sapphire and the sample was mounted at the bottom a He^3 evaporator in our cryostat.

The thermometers used were 33 ohm, 1/4 watt Allen-Bradley carbon resistors. These carbon resistors were calibrated against He^4 vapor pressure (1.2°K–2.2°K) and He^3 vapor pressure (1.2°K–0.6°K) for each measurement using a mercury and an oil manometer. The difference in height of the manometers was measured with a cathetometer.

Heat flux applied to the heater was varied for different temperatures so as to provide temperature drops ΔT at the interface between the sapphire and indium about 10 m°K. Doubling the heater power did not affect to the Kapitza resistance. This shows that $\dot{Q}$ is proportional to ΔT linearly.

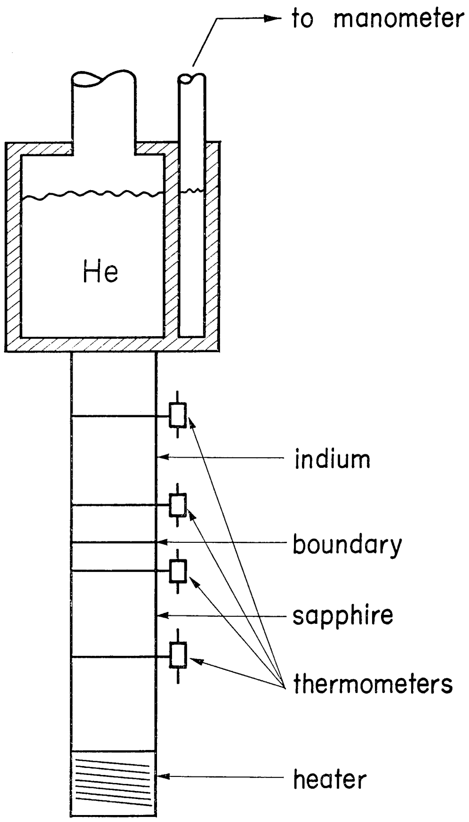

Fig. 3. A sample with thermometers and a heater.

The measurement on the normal state of the indium was done by applying current to the superconducting magnet about 17 cm long. Since our indium rod is 40 mm long, we would compensation coils on the both end of the superconducting magnet so that the magnet field is uniform along the indium rod.

The values for the Kapitza resistance R were determined from: $R=S\cdot\Delta T/\dot{Q}$, where S is the area of the interface, ΔT is the temperature jump at the interface and $\dot{Q}$ is the heat flow. Determining ΔT, we considered the effect of the long mean free path of the phonon at the sapphire.[9]

We measured the Kapitza resistance of a sample 2 in the temperature range from 1.2°K to 2.2°K two months after producing it and eight months later we did measurements in the temperature range from 0.6°K to 2.2°K. We found increase of the Kapitza resistance about 10% due to the lapse of time. We obtained results that $R_n=(27.1\pm0.2)T^{-2.58\pm0.01}$ °K cm²/W, in the normal state of indium and $R_s=(28.8\pm0.5)T^{-2.59\pm0.04}$, in the superconducting state. The typical results were shown in Fig. 4.

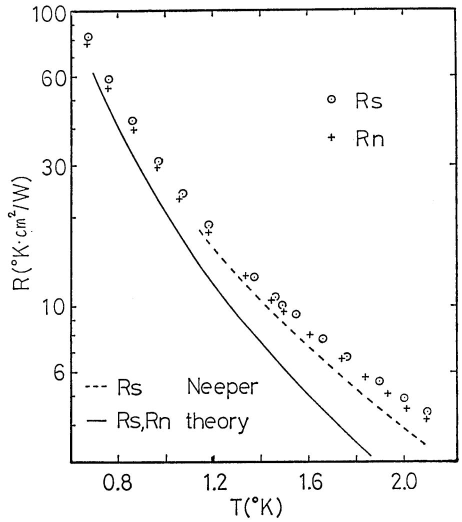

Fig. 4. Kapitza resistance on the sapphire-indium junction.

The dashed line represents Neeper's data[9] on the superconducting state (sample 2A). The solid line express theoritical values R_s and R_n. There, R_s based on the acoustic mismatch theory agrees fairly well with Nethercot[12] and R_n should coincide with R_s because of lack of surface waves, which interact with conduction electrons as calculated in § 2.

The uncertainty in the measured Kapitza resistance by thermometric error is about 4%. Also the uncertainty in the thermometer location could cause a systematic error of 2%.

In preliminary experiments, we measured the resistance of sample 1 two months after producing it. In this measurement, we obtained results that $R_s=23.7T^{-2.48}$ and $R_n=24.2T^{-2.62}$ for 1.2 to 1.9°K.

§ 4. Discussion

The estimation of the interaction of the totally reflected phonon with conduction electrons has been shown to give an appreciable contribution to the heat flow across a dielectric-metal junction. However, quantitative comparison of our calculation with experiments is difficult, because reports about measurements on the dielectric-metal junctions are quite few.

According to our estimation § 2, when the sound velocities of a dielectric are larger than those of a metal, no difference of the Kapitza resistance should be observed as the metal goes from the normal to the superconducting state. The sapphire-indium junction reported in § 3 is such a case, because the sound velocities of sapphire ($C_{1l}=12$ Km/s, $C_{1t}=6$ Km/s) are larger than the velocities of indium ($C_{2l}=3$ Km/s, $C_{2t}=1$ Km/s). This is not an exceptional case. Also, the sapphire-lead junction measured by Wolfmeyer *et al*[10] falls in the same category. The velocites of sound in dielectrics are usually larger than those of metals. We are considering as a direct check of our estimation an experiment of a dielectric-metal junction, whose velocities of sound of the dielectric are smaller.

Contrary to our estimation, a small difference between the normal and the superconducting of indium was found in our measurements. The ratio R_s/R_n is about 1.04 to 1.06. This ratio is slightly larger than 1.02 obtained by Neeper. On the other hand, Wolfmeyer *et al.* obtained results that as temperature decreases below 2°K the ratio of R_s/R_n increases above one and that it reaches approximately to 1.25 at 0.6°K and 1.3 at 0.4°K, their lowest temperature. They disagree with our's. We plotted experimental results of the three groups and our theoretical estimation in Fig. 5, whose ordinate is $^4RT^3$ instead of R. We drew curves for results by Wolfmeyer *et al*[10] without going into detail. It is a question why

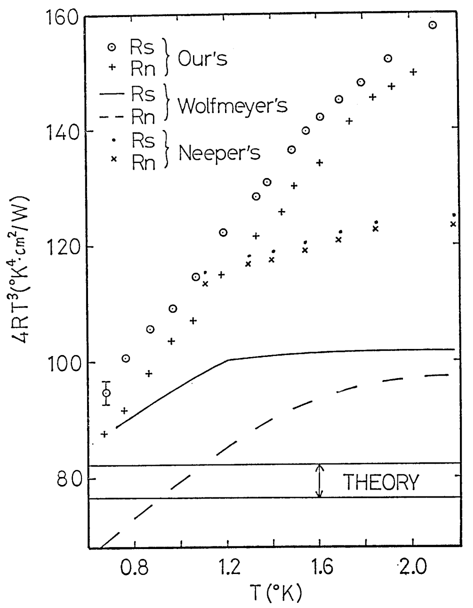

Fig. 5. Kapitza resistance ($4RT^3$ versus T).

the discrepancy takes place between three groups. The size and purity of samples are nearly same in all three but the method of fabrication of the sample by the recent Wisconsin group[10] may be different from others. They only mentioned the name of manufacturer in their paper.* We adopted the method used by Neeper, but we obtained our sapphire and indium from different sources from his. Strain at the interface seems to have some effect on the difference between R_s and R_n. However, we guess that strain is not sufficient enough to explain this difference.

Wolfmeyer *et al.* applied Andreev's theory of the Kapitza resistance between a metal and liquid helium without showing calculation to explanation of their results, but it can not be applied to this case of a metal-dielectric solid junction because of lack of surface waves interacting with conduction electrons in the metal as shown in § 2.

Qualitatively, these three results suggest that conduction electrons transfer heat in a dielectrics-metal junction by other processes than the interaction between conduction electrons and surface waves. As one of such processes, Little[5] considered a possibility that the conduction electrons interact with the periodic variation of the surface potential of a metal caused by the sound wave of liquid in the liquid-metal junction. He obtained the result that the heat transfer by this process is proportional to $H(\theta)$, where

$$H(\theta)=\int_0^{\pi/2} \Gamma^2(\Gamma')^2 \sin^2 \varepsilon_0 \cdot \cos^2\theta \cdot \sin \theta d\theta \ ,$$

and $2\varepsilon_0$ is the deviation of the phase of the reflected sound wave in liquid; Γ and Γ' are some constants due to the phase deviation of conduction electrons at the boundary. The value of ε_0 is always zero within critical angle above which the surface disturbance are created. The calculated contribution to heat transfer is four times larger than those due to interaction between surface waves and conduction electrons at the boundary between helium and a metal. He concluded that this is overestimated.

In the sapphire-indium junction, the value of

* According to our recent private communication with Dr. M. Wolfmeyer, their method of sample fabrication is as follows;

" The samples were prepared by first ultrasonically cleaning the sapphire rod. A 10 μ thick film of metal was then vacuum evaporated onto the end of the rod. The sapphire and graphite mold were next outgassed at 1000°C. Finally, ultra pure metal was vaccum cast onto one end of the sapphire rod."

ε_0 is always zero because of no such a critical angle. Thus, heat transfer by this process is zero. In order to explain our experimetal results some modification should be needed in Little's theory for such a junction or some other processes should be considered as long as we think difference of the Kapitza resistance between the two states of a metal is genuine. This is still an open question.

Finally, we point out that although Kapitza resistance obtained by all groups have less than T^{-3} temperature dependence at sapphire-indium junction in the observed temperature region, the value of R_s agrees fairly well with the theoritical value as shown in Fig. 5. The theoritical values based on the acoustic mismatch theory is estimated to lie between 19.1 and $20.5T^{-3}$ (deg cm^2/W), which agree well with Nethercot's calculation. This anbiguity of the theoretical values is caused by anisotropy of the sound velocities in indium. This agreement between the theoretical and our experimental values is much better comparing with that at helium-solid junction, where the theoritical values of R_s are few times larger than the experimental values.

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