An investigation was made into the effective application of porous heat exchangers of cylindrical shape through which fluid passes axially. On the basis of a theoretical analysis the conclusion derived was that the best thermal efficiency can be reached by the use of porous material with a large heat-exchanger surface, a high radial and low axial thermal conductivity (ie with a marked anisotropy of thermal conductivity), and a small radius of the heat exchanger operating at lower flows of cooling agent. The results of experiments carried out at helium and nitrogen temperatures are presented. These results have confirmed the high effectiveness of porous heat exchangers, even in comparison with chamber-type heat exchangers. For the temperature range from 1.5 to 300 K the heat exchangers composed of highly conductive metal nets (mesh gauge of the order of magnitude of 10^{-1} mm) stacked perpendicularly to the direction of flow of the cooling fluid, appear to be the most promising ones.

Porous heat exchangers for continuous flow helium cryostats

Z. Málek, L. Půst, and A. Ryska

In low temperature research, continuous flow cryostats[1] are often used. They allow temperatures as low as 1.5 K [2] to be reached comparatively quickly (within a period of an order of magnitude of minutes).

During short experiments or while operating flow cryostats at temperatures considerably greater than the boiling point of the cryogenic fluid, consumption is substantially lower than that of bath-type cryostats.[3]

The radiation shields were cooled and undesirable heat flows entering the experimental region were neutralized by the outflowing gaseous helium. For effective use of the cooling capacity of helium, heat exchangers having small geometrical dimensions and low hydrodynamic resistance, (which are coupled thermally with the cooled part of the cryostat) must be designed in such a way that the low temperature of the gas, which has flowed through, approaches as closely as possible, the temperature of the body being cooled. Great possibilities are offered, in this respect, by porous heat exchangers, which have been investigated and developed so far only from the point of view of the application in dilution refrigerators, ie being composed of very fine sintered grains (see Reference 4). The subject of this paper is a theoretical and an experimental analysis of cylindrical, porous heat exchangers for application incontinuousflow cryostats.

Theoretical analysis

Thermal efficiency of heat exchanger. The efficiency of a heat exchanger is defined (on the assumption of identical external conditions) as a ratio of the heat flow transferred by a real heat exchanger to the heat flow through an ideal exchanger, in which the outflowing gas has the temperature of the body being cooled. If, in the corresponding temperature interval, the dependence of the specific heat of the cryogenic fluid on the temperature is disregarded, then the efficiency

$$\eta = \frac{\overline{T}_H - T_{H0}}{T_{E1} - T_{H0}} \tag{1}$$

where $\overline{T}_H$ represents the mean temperature of helium (or other cryogenic fluid) which has passed through the heat exchanger, while $\overline{T}_H = 2/r_1^2 \int_0^{r_1} T_H(r, z_1)\, r\, dr$, T_{H0} represents the temperature of the helium entering the vessel and T_{E1} the temperature of the heat exchanger's external circumference (Fig. 1).

Thermal conductivity of porous material. At an infinite thermal conductivity, the temperature of the entire porous substance would be equal to the temperature T_{E1}. Owing to the finite thermal conductivity, an axially symmetrical

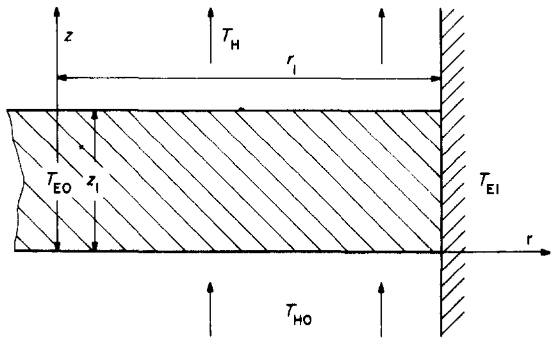

Fig. 1 Schematic section through half of a cylindrical heat exchanger, where system of coordinates, direction of gas flow and temperatures of cryogenic fluid T_{H0}, T_H and of heat exchanger circumference T_{E1} are indicated

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temperature field is set up in the heat exchanger in the stationary state.

Let us assume initially that the temperature of the heat exchanger in the axial direction is constant and that a large heat transfer between the gas and the porous substance takes place. When the temperature differences inside the heat exchanger are substantially lower than the temperature increase of the gas, determined approximately by the difference $T_{E1} - T_{H0}$, the density of heat exchange will be approximately equal throughout the heat exchanger. The heat flow through the surface defined by $r = \text{const}$ can then be expressed by means of the size of the region inside this surface. By comparing this value with the heat flow expressed by means of the temperature gradient and the thermal conductivity, it can be derived, after integration, that the behaviour of the temperature in the heat exchanger will be parabolic.

$$T_E(r) = T_{E0} + \frac{\dot{Q}}{4\pi \lambda_r z_1} \left(\frac{r}{r_1}\right)^2 \qquad (2)$$

where $\dot{Q}$ represents the total heat flow transferred, λ_r the thermal conductivity in the radial direction and T_E the temperature of the heat exchanger. From this it follows that

$$\overline{T}_H = \frac{1}{2} (T_{E0} + T_{E1}) \qquad (3)$$

From (1) and (3) we see that the following expresses a drop in efficiency $(\Delta\eta)_1$ of a real heat exchanger:

$$(\Delta\eta)_1 = 1 - \eta = (T_{E_1} - T_{E0})/2(T_{E1} - T_{H0})$$

Using (2) when $r = r_1$, with respect to (1)

$$(\Delta\eta)_1 = w\, C_p\, r_1^2\, \eta/8\lambda_r z_1 \qquad (4)$$

where w is the specific mass flow and C_p the specific heat of the cryogenic fluid. From the above relation it follows that the efficiency of the heat exchanger increases with the increasing radial thermal conductivity or with the decreasing flow or radius. Equation (4) was derived for the low temperature losses, but moreover it permits a qualitative judgement of the efficiency, even for less favourable cases.

In practice the axial thermal conductivity of a porous material is also finite and generally differs in magnitude from the radial conductivity. An examination of its effect on the efficiency of the heat exchanger will be made later.

Heat exchanging properties of the material of the heat exchanger. In order to achieve the highest possible efficiency of a heat exchanger and therefore, according to (1), temperature T_H of the gas passed through it, the highest possible heat transfer must be achieved between the proous substance and the fluid (usually represented by a cryogenic gas). The material of the heat exchanger will be considered as a quasi-continuous medium with a temperature $T_E(r, z)$ characterized by a specific surface β of the heat exchanger (the area of free surface in the volume unit of the porous material). Passing through the system axially, the gas is characterized by the temperature $T_H(r, z)$, specific mass flow w and specific heat $C_p(T_H)$.

Supposing that the heat flow passing from a solid substance to a gas is equal to the heat flow warming up the gas passing through, the situation can be described by the equation[5]

$$\partial T_H(r, z)/\partial z = (\alpha\beta/wC_p)\,[T_E(r, z) - T_H(r, z)] \qquad (5)$$

where α is the coefficient of the heat transfer. If, as in the first approximation, the complex of physical quantities $\alpha\beta/wC_p$ and temperature T_E of the porous substance in the direction z are supposed to be constant, (5) becomes an ordinary differential equation and the temperature difference between the gas and the heat exchanger will, therefore, decrease exponentially. The drop of efficiency $(\Delta\eta)_2$ due to the imperfect heat transfer, is then given by the expression

$$(\Delta\eta)_2 = \exp(-\alpha\beta z_1/w\, C_p) \qquad (6)$$

where z_1 represents the height of the heat exchanger.

When as a criterion of a good heat transfer the condition is made that temperature $T_H(r)$ of the gas passed through, should not differ from the heat exchanger temperature by more than 1% of the difference $T_E(r) - T_{H0}$, it follows that

$$\frac{\alpha\beta z_1}{wC_p} \geqslant 5 \qquad (7)$$

Calculation of temperature fields in the heat exchanger. A more accurate and complex judgement of the properties of porous heat exchangers of the type examined is offered by the calculation of the temperature fields[5] by the solution of two differential equations. One of them, (5), was mentioned earlier, the other equation expressed the effect of the heat flow, heating the gas passing through the exchanger on the temperature field in the porous substance. Its form is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \gamma \frac{\partial^2}{\partial z^2}\right) T_E(r, z) = \frac{w C_p}{\lambda_r} \frac{\partial T_H(r, z)}{\partial z} \qquad (8)$$

where the anisotropy of the porous material is characterized by the ratio of the axial and radial thermal conductivity

$$\gamma = \frac{\lambda_z}{\lambda_r} \qquad (9)$$

Introducing dimensionless temperature

$$\Theta = \frac{T - T_{H0}}{T_{E1} - T_{H0}} \qquad (10)$$

and dimensionless coordinates

$$R = \frac{r}{r_1}, \qquad Z = \frac{z}{r_1} \qquad (11)$$

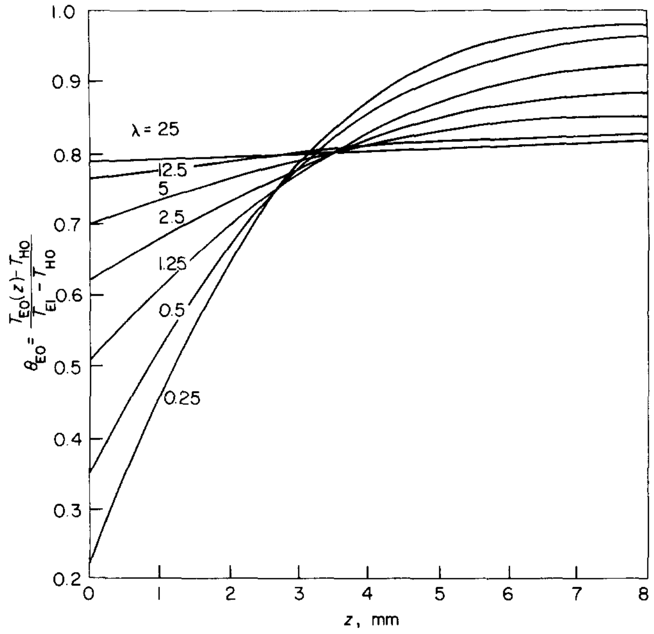

Fig. 2 Dependences of dimensionless temperature Θ_{E0} of the heat exchanger in the axis (r = 0) on the distance z from the bottom of the heat exchanger (z = 0) for different values of axial thermal conductivity λ_z. The dependences correspond to the flow w = 0.095 g s^{-1} of gas helium at α = 120 W/m^2 K, β = 7500 m^{-1}, r_1 = 25 mm, z_0 = 8 mm and λ_r = 25 W/m K ie A = 0.28 and B = 2000

equations (5) and (8) assume the form

$$\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \gamma\frac{\partial^2}{\partial Z^2}\right)\Theta_E(R,Z) = A\,\frac{\partial\Theta_H(R,Z)}{\partial Z} \quad (12)$$

and

$$\frac{\partial\Theta_H(R,Z)}{\partial Z} = B\,[\Theta_E(R,Z) - \Theta_H(R,Z)] \quad (13)$$

where the dimensionless parameters A and B denote

$$A = \frac{wC_p r_1}{\lambda_r} \quad \text{and} \quad B = \frac{\alpha\beta r_1}{wC_p} \quad (14)$$

All the physical quantities, occurring in the solved equations, can be replaced by A, B and γ.

On the assumption of constant A, B and γ, an analytical expression for the temperature fields in the porous substance and in the gas passing through can be found in the form of an infinite series of products of Bessel functions of the zero order of magnitude and a sum of three exponential functions of coordinate z. The total flow of heat transferred by the heat exchanger can also be expressed analytically. The derivation of the equations concerned and their solution have been published.[5] Using these results it is possible to prove more accurately the region of validity of the approximate relations (2), (4), (6) and (7).

Effect of anisotropy of thermal conductivity. The solution of equations (12) and (13) was found numerically[5] using twelve members of an infinite series and with a correction for the remainder of the series. Fig. 2 presents the calculated relationship between the temperature, in the centre of the heat exchanger in the axial direction $\Theta_{E0}(z)$, and various axial thermal conductivities. It will be noted that, for a markedly anisotropic porous material of a low axial thermal conductivity, the temperature of the heat exchanger for $z = z_1$ is much closer to temperature T_{E1}. Therefore, the thermal efficiency (on the assumption of a good heat transfer into the gas) is also further increased by the use of material with a markedly lower axial thermal conductivity.

Hence it follows that an estimate of the thermal efficiency drop (4) gives the upper limit of the actual situation. In a real case it is possible to increase the efficiency η considerably by the use of a suitably anisotropic material.

Effect of flow, radius and radial thermal conductivity. Fig. 3 shows the dependence of dimensionless temperature Θ_E on distance refered to the centre of the heat exchanger, on the upper, and lower plane of the porous heat exchanger. They were also calculated, using the relations solving equations (12) and (13). It is evident that with a decreasing parameter $A = wC_p r_1/\lambda_r$, the temperature of the porous substance and thus also of the gas passing through, approaches the temperature limit T_{E1} more closely, and thus also the efficiency increases. This decrease of the value A may be caused by a drop in the gas flow, the reduction of the radius of the heat exchanger, or an increase in the radial thermal conductivity of the porous substance, which is in agreement with the analysis carried out earlier. It has been found that the changes of parameter $B = \alpha\beta r_1/wC_p$ in the high value region of this parameter, according to the

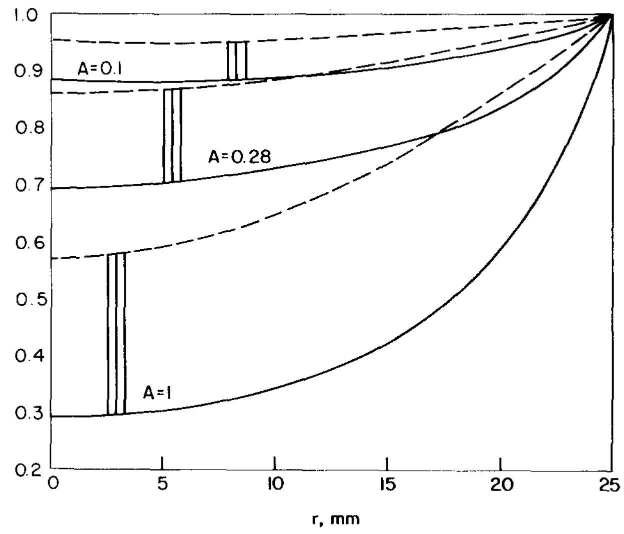

Fig. 3 Relationship between the dimensionless temperature Θ_E of the heat exchanger and the distance r from the centre for various values of the parameter A, on one hand in the lower part of the heat exchanger for z = 0 (full line), on the other hand in the upper part for $z = z_0$ = 8 mm (dotted line). In all cases r_0 = 25 mm, λ_r = 25 W (m K)$^{-1}$, λ_z = 5 W (m K)$^{-1}$, α = 120 W m^{-2} K^{-1}, β = 7500 m^{-1}. The couples of curves which correspond to the same value of A are interconnected always by three parallel lines

criterion (7), have practically no effect on the temperature field, as the heat transfer between the gas and the porous substance is always almost ideal. Therefore its effect on the temperature field can usually be disregarded.

Experimental verification of properties of heat exchangers

To verify the above mentioned calculations, test samples of heat exchangers were prepared and their properties measured at helium and nitrogen temperatures. The experiments were performed to determine the pressure gradient, the thermal conductivity in the radial direction and the overall efficiency of the heat exchangers.

Experimental arrangement. Cylindrical samples of heat exchangers E were attached in succession to the end of a stainless steel tube, P (Fig. 4), and placed in a stationary helium cryostat. The evaporated cryogenic gas passed through the tube, dipped in the cryogenic fluid, L, which stabilized the temperature of the gas, pumped by rotary vacuum pump, R, through heat exchanger, E, tube P, throttle valve V, and gas flow meter F, to the helium return distribution system, G. In the course of this process the helium inlet and outlet temperatures T_{H0} and $\bar{T}_H$ and also the temperature of the heat exchanger on its circumference, T_{E1}, and at its centre, T_{E0} were measured. The heat transfer to the circumference of the heat exchanger was provided by heating winding H.

In the central part of the heat exchanger,a closed hollow copper cylinder, C, was placed; it acted as a short circuit in the axial direction. This allowed the thermal conductivity of the porous material in the radial direction according to

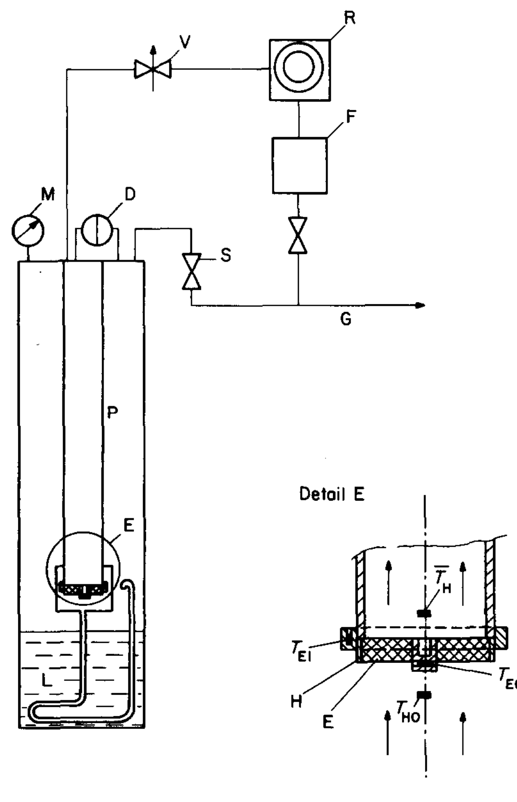

Fig. 4 Experimental arrangement for investigation of the various heat exchangers

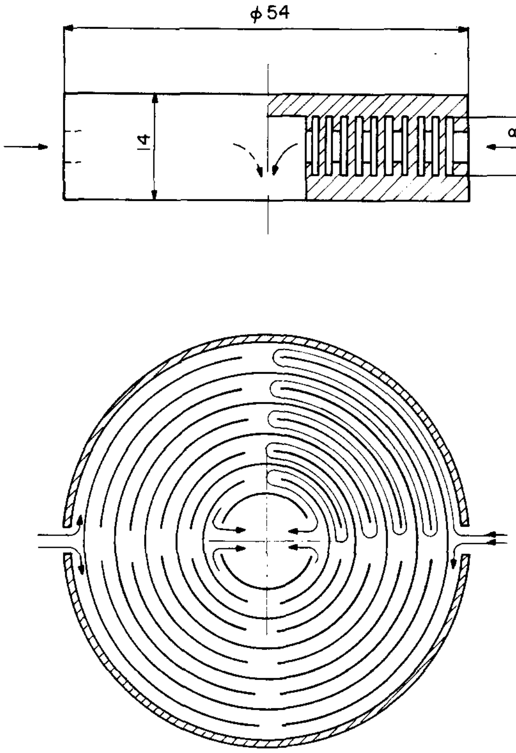

Fig. 5 A section through the chamber-type heat exchanger. The direction of cryogenic gas flow which enters through openings in the circumference, is indicated

the relation (2) to be determined. On the other hand, however, the effect of the anisotropy of the thermal conductivity of the heat exchanger was thereby practically cancelled.

Samples of heat exchangers. The first porous heat exchanger was manufactured by sintering (for a period of 3 h in a vacuum at 1243 K) fragments of electrolytic copper wire with a purity of 99.9 %, a diameter of 0.5 mm and length 2 to 4 mm. (Hereafter it will be referred to as the sintered heat exchanger.)

The second sample consisted of 35 nets of electrolytic copper, soldered at their circumference to a carrying ring. The diameter of the wire was 0.1 mm, and the edge of the mesh 0.23 mm. In both cases the radius of the heat exchanger inside the carrying ring was $r_1 = 25$ mm, the height $z_1 = 8$ mm and the diameter of the cylinder, C, in the centre was 11 mm. (Referred to as the net heat exchanger.)

For comparison, the third copper heat exchanger of the chamber-type was investigated (chamber-type heat exchanger). It has an active region of the same outer dimensions as the porous heat exchangers (Fig. 5).

Thermal conductivity. From the dependences between the total flow $\dot{Q}$ of the heat transferred and the difference of the temperatures in the centre T_{E0} and on the circumference of the heat exchanger T_{E1}, the radial thermal conductivity λ_r of the porous material can be determined by (2). In the region of nitrogen temperatures, where the thermal conductivity of copper is approximately constant, the following ratio was derived

$$\lambda_r = \frac{\dot{Q}}{4\pi z_1 (T_{E1} - T_{E0})} \qquad (15)$$

In the region of helium temperatures the linear dependence of λ_r on temperature has to be considered. Then the thermal conductivity at 4.2 K can be determined from the relation

$$\lambda_r(4.2) = \frac{4.2}{T_E} \frac{\dot{Q}}{4\pi z_1(T_{E1} - T_{E0})} \quad (16)$$

where $\overline{T}_E$ is the mean temperature of the porous material in the respective experimental point.

The total flow $\dot{Q}$ of the heat transferred from the heat exchanger was determined from the temperature increase of the cooling gas passing through the heat exchanger. Temperatures T_E and T_H were measured by means of Allen-Bradley carbon resistors (270 ohms, 1/8 W), calibrated at the appropriate temperature intervals by means of germanium and platinum thermometric sub-standards.

From the measured values of $\dot{Q}$, T_{E1} and T_{E0} the radial thermal conductivity of the net heat exchanger was determined in the region of nitrogen and helium temperatures, and in the case of the sintered heat exchanger in the region of nitrogen temperatures. The results are presented numerically (including standard deviations) in Fig. 6. From these results it follows that the sintered heat exchanger has, at nitrogen temperatures, approximately one third of the thermal conductivity of the net heat exchanger. Since, at nitrogen temperatures, the purity of the porous copper material and other lattice defects have no substantial effect upon the thermal conductivity, it is obvious that of the two samples the net heat exchanger is more suitable from the point of view of heat transfer.

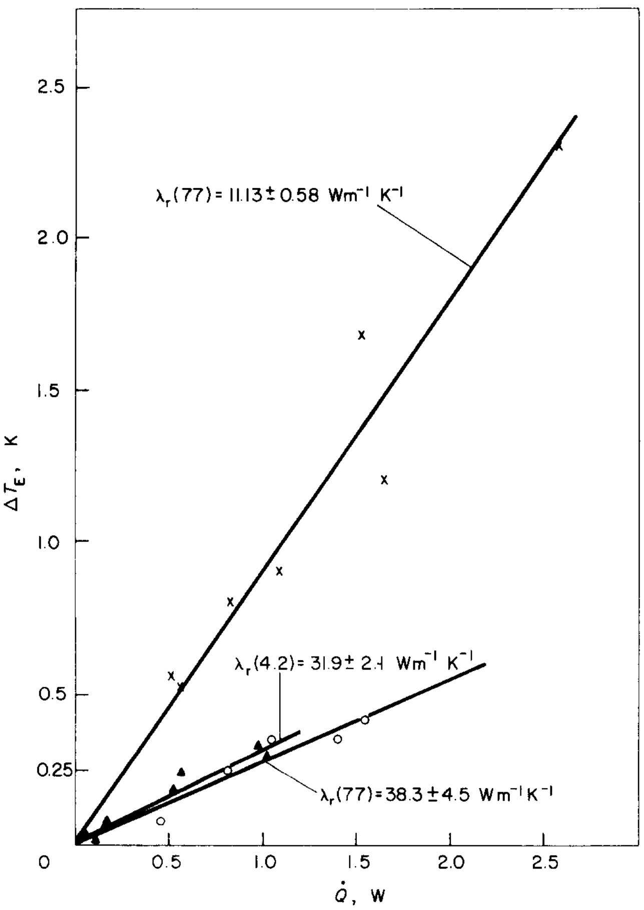

Fig. 6 Measured dependences of the temperature difference between the circumference and centre of the porous heat exchanger, on the total heat flow transferred by the heat exchanger. The indicated values of the radial thermal conductivity have been calculated according to (15) and (16). + — sintered heat exchanger (cooled by N_2); ○ — net heat exchanger (cooled by N_2); ▲ — net heat exchanger (cooled by He)

Thermal efficiency. The thermal efficiency η_{exp} of the samples of heat exchangers was determined from the experimental values of temperatures according to (1), a correction being made for the effect of the partial reverse flow in the space of the gas, which was flowing out of the heat exchanger and gradually heated in tube, P (Fig. 4).

In Table 1 the results are presented for the three types of heat exchanger investigated.

In the last column, the average values of efficiency are presented, calculated according to the relation (4) where the average values of the quantities concerned were substituted. For the values of thermal conductivity the data given in Fig. 6 were substituted.

According to the results presented in Table 1, porous heat exchangers (with the dimensions and materials used) reach the region of helium and nitrogen temperatures an efficiency close to unity. The chamber-type heat exchanger has an efficiency 5 to 9 % lower. This difference is predominantly caused by a larger area of active heat exchanging surface of the porous heat exchangers ($\beta \approx 10^4 \text{m}^{-1}$) in comparison with the chamber-type exchanger ($\beta \approx 5 \times 10^2 \text{m}^{-1}$). For a comparison of porous heat exchangers, the last column of values is decisive, as these values of efficiency are not influenced by the error in T_H caused by the return flow of the cryogenic gas in tube P (see Fig. 4). Hence it follows that a higher efficiency is reached by a net heat exchanger, even in the case when the effect of anisotropy of the thermal conductivity is cancelled. If a net heat exchanger is produced, in which the system of nets in the central part is not axially thermally short-circuited, the efficiency will be further increased.

For both types of porous heat exchangers the resulting left side of (7), which characterizes the heat transfer from the heat exchanger to the cryogenic fluid, is at least one order of magnitude higher than demanded by the above mentioned condition at all flows. Hence, the use of the finer porous material does not appreciably improve the efficiency of the heat exchanger, but causes an increase in its hydrodynamic resistance. This conclusion is also supported by the fact that the $\overline{\eta}$ values, calculated assuming ideal heat transfer, (presented in the last column of Table 1) agree relatively well with the efficiencies determined experimentally. Further improvement of the efficiency can be achieved, apart from the utilization of the anisotropy of the thermal conductivity, by an increase in the radial thermal conductivity of the heat exchanger, as follows from the relation (4). This can be done either by the use of material of a higher specific thermal conductivity (extremely pure copper or silver) or by increased sintering of grains, or the use of nets with finer meshes and thicker wires.

Pressure gradients. To compare the hydrodynamic resistance of various types of heat exchangers, the pressure gradients within the individual heat exchangers were measured for various gas flows by means of a differential pressure gauge[7] with a sensitivity of 1 Pa.

Table 1. Thermal efficiency of $\bar{\eta}$ of heat exchangers under investigation

Type of heat exchanger	Cryogenic gas	Temperature interval of heat exchanger (K)	Mass flow interval ($g\ s^{-1}$)	Number of measurements	$\bar{\eta}_{exp}$ (measured)	Standard deviation	$\bar{\eta}$ (calculated)
Sintered-type	N_2	77–90	0.06–0.20	7	0.95	0.04	0.94
Net-type	N_2	77–90	0.06–0.20	5	0.95	0.03	0.98
Net-type	He	4.2–8.0	0.04–0.13	7	0.97	0.08	0.96
Chamber-type	N_2	77–90	0.05–0.16	13	0.89	0.03	–

Table 2. Pressure gradients on chamber-type heat exchanger

Mass flow ($g\ s^{-1}$)	0.05	0.10	0.15
Pressure gradients with nitrogen (Pa)	125	490	960
with helium (Pa)	305	620	1180

The pressure gradients were closely investigated on the chamber-type heat exchanger, the diagram of which is shown in Fig. 5. Table 2 presents the experimental results for applied cooling by gaseous nitrogen and helium, the temperature of which at the heat exchanger inlet was always approximately equal to the boiling point (at atmospheric pressure) of the cryogenic fluid concerned. The pressure gradients measured on the sintered and net heat exchangers were two orders of magnitude lower and there was no marked difference between the resistance of the sintered and the net heat exchanger.

Conclusion

In this paper an analysis,has been made of the heat transfer conditions in a cylindrical heat exchanger, into which heat enters radially through the circumference and is transferred axially by the flowing cryogenic fluid. The effects of the flow of cryogenic fluid, of the radius and height of the heat exchanger and of its radial thermal conductivity, of the heat transfer coefficient and of the heat exchanging surface, have been investigated. Their action may be expressed quantitatively by (4) and (6). The effects of the magnitude of the gas flow, of the radius of the heat exchanger and of its thermal conductivity are shown graphically in Fig. 3. The effect of anisotropy of the thermal conductivity of the heat exchanger in the radial and axial directions is considered quantitatively and the results are presented in Fig. 2.

The results of the theoretical analysis were verified on two porous and one chamber-type heat exchanger. On the porous type, the radial thermal conductivity, the thermal efficiency and the hydrodynamic resistance were measured during cooling by gaseous helium (and also nitrogen) in temperature intervals of 4.2 to 8 K (77 to 90 K) at mass flows within a range of 4×10^{-2} to 2×10^{-2} g s^{-1}. The results prove that both porous heat exchangers reach, within the full range of the conditions mentioned, a thermal efficiency above 94% (Table 1), even with an unsuitable design (axial thermal short circuit in centre part), which was necessitated by the requirement of measurement of the radial thermal conductivity. The results of the measurement of the thermal conductivity of the porous heat exchangers are presented in Fig. 6. The results of the measurement of the hydrodynamic resistance are shown, for the chamber-type heat exchanger (Fig. 5) in Table 2. With porous heat exchangers the hydrodynamic resistance measured were, under the same conditions, two orders of magnitude lower.

We conclude that, for the design of continuous flow cryostats intended for a temperature range of 1.5 to 300 K, heat exchangers composed of the order of ten layers of metal nets of highly conductive material (eg silver or pure copper of approximately 0.2 mm wire diameter and 0.1 mm size of mesh) welded only to the circumference of the heat exchanger from which heat is to be removed, are particularly suitable. The efficiency of these heat exchangers at an outside diameter of 5 cm and a height of 0.8 cm at specific mass flows amounting to 10^{-1} kg $m^{-2} s^{-1}$ almost invariably exceeds 95% at a pressure gradient on the heat exchanger of an order of magnitude 10 Pa or lower.

Only somewhat less favourable results can be achieved by means of heat exchangers manufactured by sintering of fragments of analogous metallic materials.

Chamber-type heat exchangers which usually have, at higher efficiencies, a hydrodynamic resistance at least one order of magnitude higher, are also usable, supposing that their design provides relatively high value of active heat exchanging surface ($\beta > 10^3 m^{-1}$).

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