PAUL SCHERRER INSTITUT

Memorandum

Datum: August 15, 2012

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Meissner Response for YBCO under Light Irradiation

YBCO shows a photo-persistent shift of T_c. The question is, how could the Meissner response be modified, *i.e.* how is the charge distributed in the YBCO after irradiation? The London equation, assuming a constant superfluid density, n_S, up to the very surface is

$$\frac{\mathrm{d}^2 B}{\mathrm{d}z^2} = \frac{1}{\lambda^2} B \propto n_\mathrm{S} B \tag{1}$$

The connection between n_S and λ is

$$\lambda^2 = \frac{m}{\mu_0 e^2 n_\mathrm{S}} \tag{2}$$

The solution of Eq.(1) for a flat semi-infinite sample is

$$B(z)/B_\mathrm{ext} = e^{-z/\lambda} \tag{3}$$

For a thin film with thickness $2t$ the solution is

$$\frac{B(z)}{B_\mathrm{ext}} = \frac{\cosh[(t-z)/\lambda]}{\cosh[t/\lambda]} \tag{4}$$

However, the light absorption is $\propto \exp(-z/z_\mathrm{light})$, where z_light will depend on the photon energy and the material. In principle there are two conceivable scenarios: (i) the photo generated charge carriers will diffuse freely throughout the YBCO, in which case n_S will slightly increase ($n_\mathrm{S} \to \tilde{n}_\mathrm{S} > n_\mathrm{S}$) and hence $\tilde{\lambda} < \lambda$. The functional form of $B(z)$ is not influenced in this case, only λ changes. (ii) the photo generated charge carriers are "pinned" to the layer in which they where generated, *i.e.* they can freely move parallel to the interface but *not* perpendicular to it. In this case the superfluid density will have a spatial depends and hence the Meissner response $B(z)$ will be modified. Assuming that the superfluid density $n(z)$ has the following form

$$n(z) = n_\mathrm{S} + n_\mathrm{p} e^{-z/z_0}, \tag{5}$$

with n_p the photo-induced superfluid density with a range z_0 (it is not clear that $z_0 = z_\mathrm{light}$), The London equation will be modified to

$$\frac{\mathrm{d}^2 B}{\mathrm{d}z^2} = \left[\frac{1}{\lambda^2} + \frac{1}{\lambda_\mathrm{p}^2} e^{-z/z_0}\right] B. \tag{6}$$

Thin Film

Eq.(6) can be solved (by the help of Mathematica) for the thin film boundary conditions ($B(z = 0)/B_\mathrm{ext} = 1$, and $B(z = D)/B_\mathrm{ext} = 1$). The solution is

$$\frac{B(z)}{B_{\rm ext}} = \frac{1}{N}\Big[I_{\nu_-}(\nu_{\rm p}\exp[-z/(2z_0)])\left\{I_{\nu_+}(\nu_{\rm p}) - I_{\nu_+}(\nu_{\rm p}\beta)\right\} - I_{\nu_+}(\nu_{\rm p}\exp[-z/(2z_0)])\left\{I_{\nu_-}(\nu_{\rm p}) - I_{\nu_-}(\nu_{\rm p}\beta)\right\}\Big], \tag{7}$$

with

$$\begin{aligned} N &= I_{\nu_-}(\nu_{\rm p}\beta)I_{\nu_+}(\nu_{\rm p}) - I_{\nu_-}(\nu_{\rm p})I_{\nu_+}(\nu_{\rm p}\beta) \\ \nu_\pm &= \pm\frac{2z_0}{\lambda} \\ \nu_{\rm p} &= \frac{2z_0}{\lambda_{\rm p}} \\ \beta &= \exp(-t/z_0) \qquad t = D/2, \quad D: \text{ film thickness} \end{aligned}$$

with $I_\nu(z)$ be modified Bessel function of first kind[1]. $I_\nu(z)$ can be written as

$$I_\nu(z) = (z/2)^\nu \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{k!\,\Gamma(\nu+k+1)}, \tag{8}$$

with the asymptotic behavior ($z \to 0$, ν fixed)

$$I_\nu(z) \simeq \frac{(z/2)^\nu}{\Gamma(\nu+1)} \qquad (\nu \neq -1, -2, \ldots)$$

Figs.1-4 illustrated the behavior. Fig.1 shows the modification of $B(z)$ for the case where z_0 is rather short. For this case the photo-induced $B(z)$ shows an almost parallel shift only compared to the London case. Only at small z-values there are not parallel anymore. This is very close to the observed situation. In the experiment, the small z-values were not reachable. The fitting suggest, that the "dead layer" shrinks, which is consistent with Fig.1 when parallel extrapolating towards $z \to 0$. Fig.2 shows $B(z)$ over the full film thickness, showing that on the "substrate" side, the deviations of the $B(z)$'s are small, as expected. The given parameters are not very realistic, since $\lambda/\lambda_{\rm p} = 2$, meaning that there is a tremendous increase of the superfluid density very close to the surface. This parameters are rather chosen to make the case!

Fig.4 shows the situation for $z_0 = 0.3$, *i.e.* 3 times larger z_0 compared to Figs.1-2. In order to be able to compare them, the integral induced photo superfluid density

$$\int_0^\infty \frac{1}{\lambda_{\rm p}^2} e^{-z/z_0} {\rm d}z = \frac{z_0}{\lambda_{\rm p}^2} \tag{9}$$

is kept constant, hence $\lambda_{\rm p,2} = \lambda_{\rm p,1}\sqrt{z_{0,2}/z_{0,1}}$. It is interesting to note that for larger z_0 the vacuum side slope between the two curves is changing rather than finding a "parallel" offset. This is what one would expect since for $z_0 \to \infty$ and keeping $z_0/\lambda_{\rm p}^2$ constant, this would just result into case (i).

[1] see M. Abramowitz and I.A. Stegun "Handbook of Mathematical Functions", p. 375ff, Chapter: Modified Bessel Functions I and K.

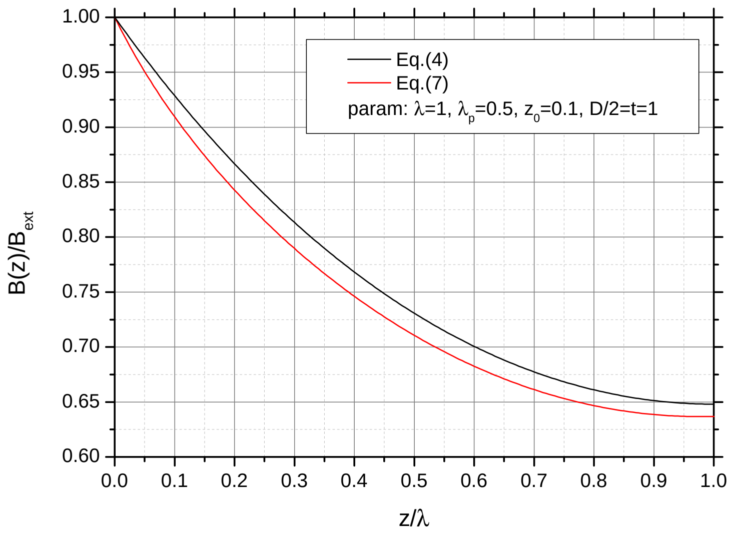

Figure 1: $B(z)$ for half the film only. Parameters: $\lambda = 1$, $\lambda_\mathrm{p} = 0.5$, $z_0 = 0.1$, $t = 1$.

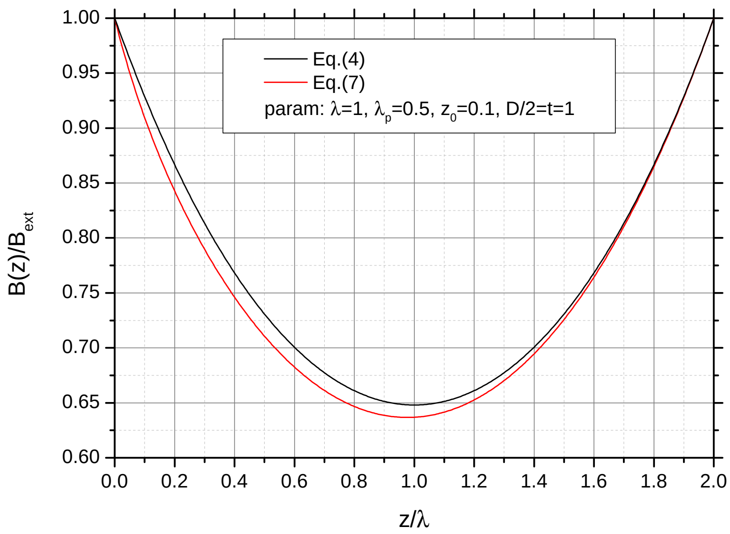

Figure 2: $B(z)$ for full film thickness. Parameters: $\lambda = 1$, $\lambda_\mathrm{p} = 0.5$, $z_0 = 0.1$, $t = 1$.

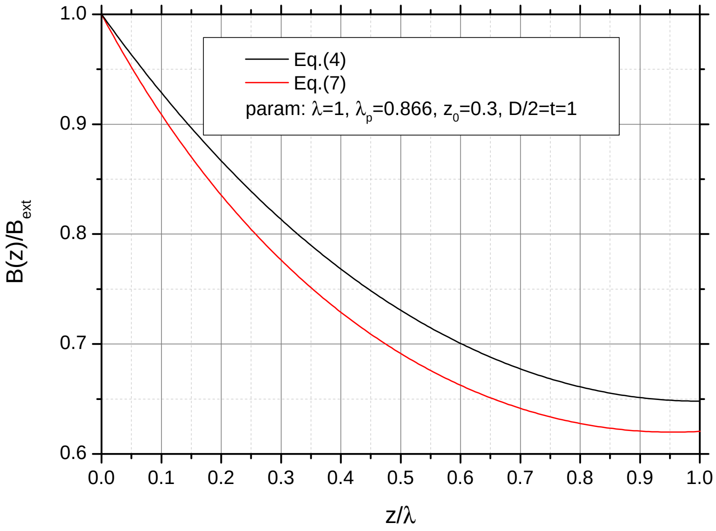

Figure 3: $B(z)$ for full film thickness. Parameters: $\lambda = 1$, $\lambda_{\mathrm{p}} = 0.5 \times \sqrt{3}$, $z_0 = 0.3$, $t = 1$.

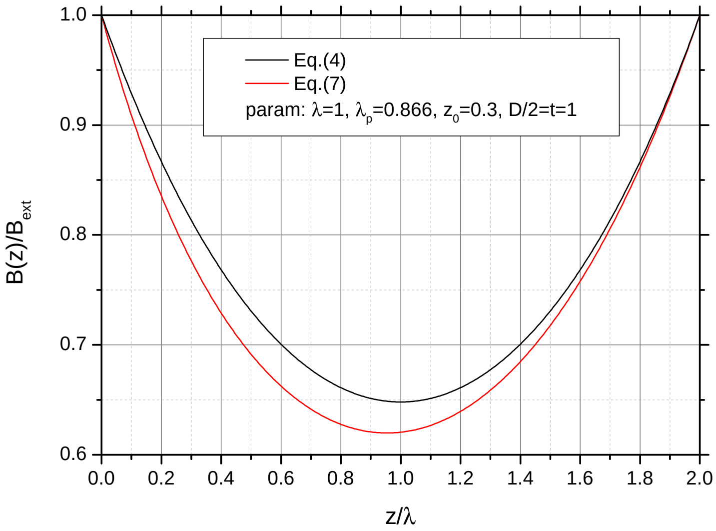

Figure 4: $B(z)$ for full film thickness. Parameters: $\lambda = 1$, $\lambda_{\mathrm{p}} = 0.5 \times \sqrt{3}$, $z_0 = 0.3$, $t = 1$.

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Semi-Infinite Sample

For a semi-infinite sample, the solution of Eq.(6), for the boundary conditions $B(z=0)/B_{\rm ext} = 1$ and $B(z)/B_{\rm ext} \to 0$ for $z \to \infty$, is

$$\frac{B(z)}{B_{\rm ext}} = \frac{I_{\nu_+}(\nu_{\rm p}\sqrt{\exp(-z/z_0)})}{I_{\nu_+}(\nu_{\rm p})}, \tag{10}$$

with

$$\begin{aligned} \nu_+ &= \frac{2z_0}{\lambda} \\ \nu_{\rm p} &= \frac{2z_0}{\lambda_{\rm p}} \end{aligned}$$

Eq.(10) needs to be compared to the usual London screening $B(z)/B_{\rm ext} = \exp(-z/\lambda)$.
Fig. 5 shows the $B(z)/B_{\rm ext}$ for Eq.(10), for the same parameters as used for the thin film.

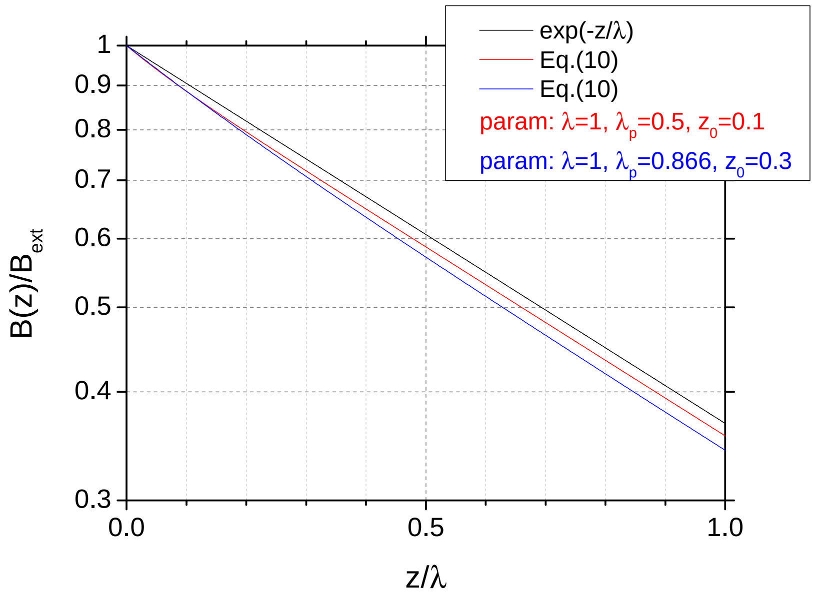

Figure 5: $B(z)$ for a semi-infinte sample. Black: $\exp(-z/\lambda)$, Red and Blue from Eq.(10). Parameters: Red: $\lambda = 1$, $\lambda_{\rm p} = 0.5$, $z_0 = 0.1$. Blue: $\lambda = 1$, $\lambda_{\rm p} = 0.5 \times \sqrt{3}$, $z_0 = 0.3$

Useful Relations

Most numerical libraries only implement $I_n(z)$ for $n > 0$. Hence Eq.(7) couldn't be calculated. The following relation solves this problem (see M. Abramowitz and I.A. Stegun "Handbook of Mathematical Functions", 9.6.2, p.375):

$$I_{-n}(z) = I_n(z) + \frac{2}{\pi}\sin(n\pi)\,K_n(z),$$

where $K_n(z)$ is the modified Bessel function of 2$^{\rm nd}$ kind.