On the electric field and muon-paths inside the thermal shield.

One of the ingredients for an understanding of the implantation-energy dependence of the signal of the LEM setup is the knowledge of the path of the muons inside the thermal shield.
The electric potential and thus the electric field inside the thermal shield on the Konti-1,2 and MANGO cryostats can be straightforwardly calculated using the relaxation method. Based on this potential, a simple simulation can be made of the muons entering the shield area parallel to the cylindrical axis.

The electric potential in a charge-free space has to obey the Laplace equation, $\Delta V = 0$.
In dimension one this reduces to

(1.1) $$\frac{\partial^2 V}{\partial x^2} = 0\,,$$

which has as solution a straight line between the two boundary condition.

In dimension two the equation reads:

(1.2) $$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Digitized eq. (1.2) reads:

(1.3) $$\frac{V(i-1,j)+V(i+1,j)-2V(i,j)}{dx} + \frac{V(i,j-1)+V(i,j+1)-2V(i,j)}{dy} = 0$$

or

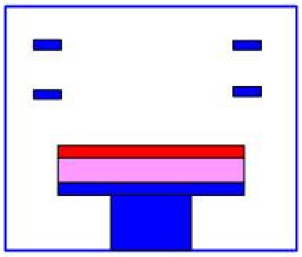
(1.4) $$V(i,j) = \frac{V(i-1,j)+V(i+1,j)+V(i,j-1)+V(i,j+1)}{4}$$

or, each $V(i,j)$ equals the average of its four nearest neighbours.

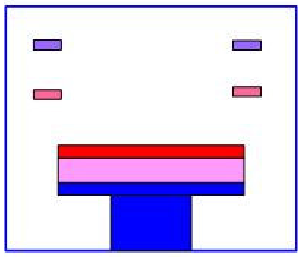
That is the ground for the relaxation method: define the values of V on the boundaries and apply eq. 1.4 to each cell and repeat (e.g. 1M times) until a stable solution is reached.

The thermal shield area is cylinder symmetric, therefore, using cylinder coordinates leaves us with ρ and z as variables. The cylinder character of the coordinates change eq. 1.4, since the Laplacian in cylinder coordinates reads:

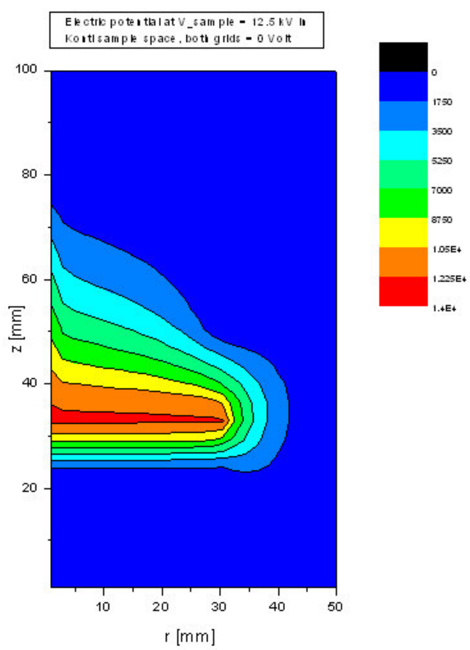
(1.5) $$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

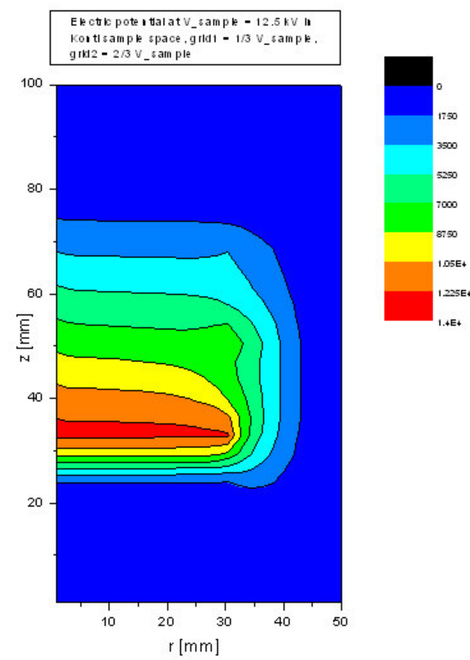


The configuration is depicted in the left hand figure. The colors indicate the electric potential, red being 12.5 kV (= V_{sample}), blue 0.0 V, respectively. The top panel shows the situation where the grids (or rings) are on zero potential, in the bottom panel the lower grid has two-thirds of V_{sample} and the upper grid one-third of V_{sample}.



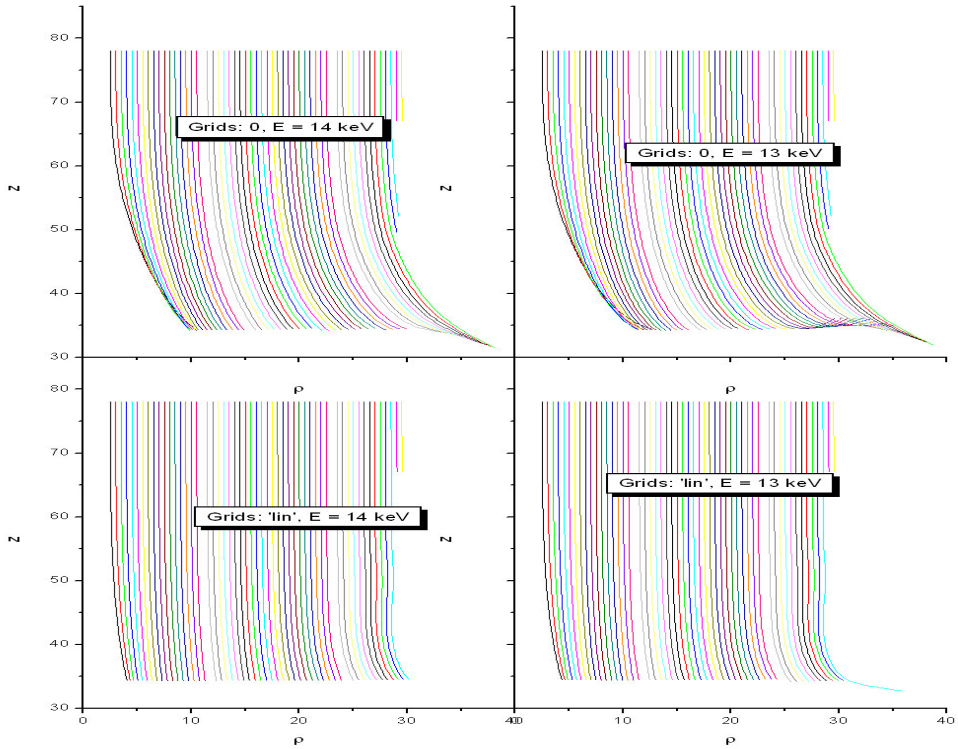
The electric potentials are shown below:





The electric fields are, of course, given by $-\nabla V$.

The muonpath are now found via $ds = v_0 dt + \frac{1}{2} a \left(dt \right)^2$, *with* $a = qE/m$ and repeating this until the muon hits a boundary. The paths obtained are shown in the following figures:



As can be seen, more muons will hit the sample, independent of transport energy, is the grids are on 1/3 and 2/3 V_{sample}. The times at which the muon hits the sample after entering the thermal shield is given in the following table:

	Grids = 0	Girds linear
13 keV	12.7	14.1
14 keV	11.1	12.2

In other words, no significant delay due to the grids at 1/3 and 2/3 voltage !!.

Conclusion: using the grids should have an advantage and hopefully reduces the "instrument effect"

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