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Memorandum

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London theory including the reduction of the orderparameter at the interface

The 2[nd] London equation for a semiinfinite interface has the form

$$\frac{\mathrm{d}^2 A}{\mathrm{d}z^2} = \frac{4\mu_0 e^2 |\Psi_\infty|^2}{m} A(z) = \frac{1}{\lambda_{\mathrm{L}}^2} A(z). \tag{1}$$

where $|\Psi_\infty|^2$ is the superfluid density, assumed to be constant in the London model.
In the following $A' = \mathrm{d}A/\mathrm{d}z$ will be used. The boundary conditions are deduced in the following way. The London gauge requires $\nabla \cdot A = 0$, hence $A(z) = A_0 + B_0 z, \quad \forall z < 0$, where B_0 is the externally applied magnetic field. In order to merge the solutions at the interface $\Longrightarrow A_0 = -\lambda_{\mathrm{L}} B_0$ [1], and therefore the boundary conditions are

$$A(0) = -\lambda_{\mathrm{L}} B_0 \tag{2}$$

$$A'(0) = B_0 \tag{3}$$

which results in the well known

$$A(z) = -\lambda_{\mathrm{L}} B_0 \exp(-z/\lambda_{\mathrm{L}}) \tag{4}$$

and therefore

$$B(z) = A'(z) = B_0 \exp(-z/\lambda_{\mathrm{L}}) \tag{5}$$

Within Ginzburg-Landau theory, one can show that the orderparameter $\Psi_\infty f(z)$ will decrease when reaching the interface [1, 2]. Under the assumption that $\lambda_{\mathrm{L}} \to 0$ an finds a functional form of $f(z)$

$$f(z) = \tanh\left[\frac{z}{\sqrt{2}\xi(T)}\right] \tag{6}$$

where $\xi(T)$ is the Ginzburg-Landau coherence length. Utilizing this result, Eq.(1) can be rewritten as

$$A''(z) = \left[\frac{f(z)}{\lambda_{\mathrm{L}}}\right]^2 A(z). \tag{7}$$

This differential equation is a gross oversimplification, since it ignores that non-local effects should be taken into account and further λ_{L} is finite. Still it is going to be interesting to see the outcome of Eq.(7) to get a feeling how the magnetic field will penetrate the superconductor. Unfortunately, Eq.(7) can only be solved numerically. I used `Mathematica` for this purpose. The used code is given in Sec.A. Fig.1 shows a typical $B(z)$ for $\xi(T) = 1$ and $\lambda_{\mathrm{L}} = 0.3$. As

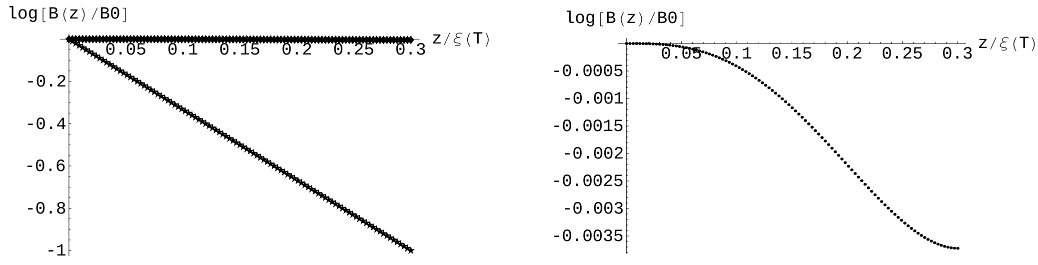

Figure 1: Left graph: Comparison between the exponential decay and the result of Eq.(7) for $\xi(T) = 1$ and $\lambda_{\mathrm{L}} = 0.3$. Right graph: Only $B(z)$ as from Eq.(7) (notice the scale!).

can been seen the magnetic field is penetrating almost unhindered the superconductor. Since this crude model is maximal valid for values $z < \lambda_{\mathrm{L}}$ only this part is shown. The trend is exactly as expected: The reduction of the superfluid density close to the surface decreases the screening and hence the field penetrates easily. That it is such a drastic effect is only since the model ignores non-local effects and therefore neither $f(z)$ (which would have to be estimated self-consistently) nor $B(z)$ can be correct and only can show a trend.

A Mathematica Code to implement Eq.(7)

```
In[1]:= f = Tanh[z/Sqrt[2]/xi]
In[2]:= param = {xi -> 1, lambdaL -> 0.3}
In[3]:= deq = {A’’[z] - (f/lambdaL)^2 A[z] == 0 /. param, A[0] == -lambdaL /. param, A’[0] == 1}
In[4]:= solution = NDSolve[deq, A, {z, 0, lambdaL /. param}]
In[5]:= Plot[{A[z] /. solution, -lambdaL Exp[-z/lambdaL] /. param}, {z, 0,
          lambdaL /. param}, PlotRange -> All]
In[6]:= Plot[{A’[z] /. solution, Exp[-z/lambdaL] /. param}, {z, 0, lambdaL /. param},
          PlotRange -> All]
In[7]:= BB = Table[{z, (Log[A’[z] /. solution])[[1]]}, {z, 0, lambdaL /. param,
          lambdaL/100 /. param}];
In[8]:= EE = Table[{z, Log[Exp[-z/lambdaL] /. param]}, {z, 0, lambdaL /. param,
          lambdaL/100 /. param}];
In[9]:= << Graphics‘MultipleListPlot‘
In[10]:= MultipleListPlot[BB, EE, PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 0, 0]}]
```

References

[1] C.P. Poole Jr., H.A. Farach, and R.J. Creswick, *“Superconductivity”*, Academic Press (1995), p.128ff.

[2] P.-G. deGennes, “Superconductivity of Metals and Alloys”, Addison-Wesley (1989), p.177ff.