# Thermometry by Arrays of Tunnel Junctions

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We show that arrays of tunnel junctions between normal metal electrodes exhibit features suitable for primary thermometry in an experimentally adjustable temperature range where thermal and charging effects compete. $I$-$V$ and $dI/dV$ vs $V$ have been calculated for two junctions including a universal analytic high temperature result. Experimentally the width of the conductance minimum in this regime scales with $T$ and $N$, the number of junctions, and its value (per junction) agrees with the calculated one to within 3% for large $N$. The height of this feature is inversely proportional to $T$.

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Discussions on tunnel junction arrays typically concentrate on their properties in the regime where charging energy $E_c$ well dominates over the thermal energy $k_BT$ [1,2]. In this case, the arrays exhibit extreme Coulomb blockade with very little variations due to temperature changes. If we, however, look at the opposite extreme, where $E_c \ll k_BT$, we find a *primary thermometer,* whose dynamic temperature range can be tailored by choosing the proper dimensions of tunnel junctions. In this high $T$ regime, the remarkable property of these arrays is that $V_{1/2}$, the full width at half maximum of the charging peak, i.e., of the conductance drop, divided by temperature is a universal number for all the arrays with the same number of junctions in series, $N$, provided that the variation in junction parameters is not excessively large. We show this theoretically for a symmetric double junction array with a result $eV_{1/2}/2k_BT = 5.439\ldots (\equiv v_{1/2,0})$. We confirm this result experimentally and find that the width scales with $N/2$. In addition to $V_{1/2}$ being proportional to $T$, the relative change of conductance with bias is inversely proportional to $T$ due to the conserved area. The $I$-$V$ curve of the array can be calculated also at low temperatures, but here the temperature dependence of this feature weakens, and the undesirable charge sensitivity sets in.

Let us consider the simplest case of a double junction array in an idealized symmetric configuration, as shown in Fig. 1(a). The two tunnel junctions in series both have a tunnel resistance $R_T$, and their capacitance equals $C$. The island in between has a ground capacitance $C_0$. The four tunneling rates of the problem are $\Gamma_i^{\pm}$, $i = \{1,2\}$. The total capacitance to charge the island $C_\Sigma$ is given by $C_\Sigma = 2C + C_0$. One end of the chain is biased at $+V/2$ and the opposite end at $-V/2$. The charging energy of the island equals $E_c = e^2/2C_\Sigma$.

Tunneling in a chain of junctions is typically theoretically treated by Monte Carlo simulations [3,4], whereas the problem of a double junction system can be solved at any temperature, even when asymmetries are introduced by a straightforward method as shown below. Our derivation is a followup of the "orthodox theory" [1]. The tunneling rates are fully determined by the free energy difference of the states before and after a tunneling event. This is given by

$$-\Delta F = (e/2)[(\phi_2 + \phi_2') - (\phi_1 + \phi_1')]. \qquad (1)$$

Here $\phi_1$ and $\phi_2$ are the potentials of the electrodes from and to which the electron tunnels, respectively, and the nonprimed and primed potentials are those before and after tunneling, respectively. Let us denote the potential of the island with $n$ extra electrons ($n$ may be negative as well) by $\phi(n)$ [see Fig. 1(a)]. We have

$$\phi(n) = ne/C_\Sigma \qquad (2)$$

due to charge conservation. The tunneling rate for a particular event is given by

$$\Gamma = (e^2R_T)^{-1}\frac{\Delta F}{1 - \exp(-\Delta F/k_BT)}. \qquad (3)$$

To calculate the $I$-$V$ curve we use the relation

$$I_i = e\sum_{n=-\infty}^{\infty}\sigma(n)[\Gamma_i^+(n) - \Gamma_i^-(n)], \qquad (4)$$

where $I_i$ is the current through the $i$th junction, $i = \{1,2\}$, and $\sigma(n)$ is the probability of finding $n$ extra electrons on the island. To find $\sigma(n)$ we use the master equation, which reads

$$\begin{aligned}&[\Gamma_1^+(n-1) + \Gamma_2^-(n-1)]\sigma(n-1)\\ &\quad -[\Gamma_1^+(n) + \Gamma_1^-(n) + \Gamma_2^+(n) + \Gamma_2^-(n)]\sigma(n)\\ &\quad + [\Gamma_1^-(n+1) + \Gamma_2^+(n+1)]\sigma(n+1) = 0 \qquad (5)\end{aligned}$$

in a steady situation. Symmetry yields $I_1 = I_2 \equiv I$ in Eq. (4) if we set $\sigma(-n) = \sigma(+n)$. A straightforward rigorous way of calculating $I$-$V$ and $dI/dV \equiv G$ vs $V$ is thus enabled by Eqs. (1)–(5).

In the high temperature limit, $u \equiv (e^2/C_\Sigma)/k_BT \ll 1$, fully analytic expressions can be obtained for the conductance curve vs bias voltage. Taking $u$ as the small parameter in the expansion of $\Gamma_i^{\pm}(n)$ we arrive at

$$\begin{aligned}\Gamma_i^+(n) - \Gamma_i^-(n) &= (k_BT/e^2R_T)\\ &\quad\times(v + \{[f(v) - f(-v)]/2 - n\}u) + \cdots\end{aligned} \qquad (6)$$

  

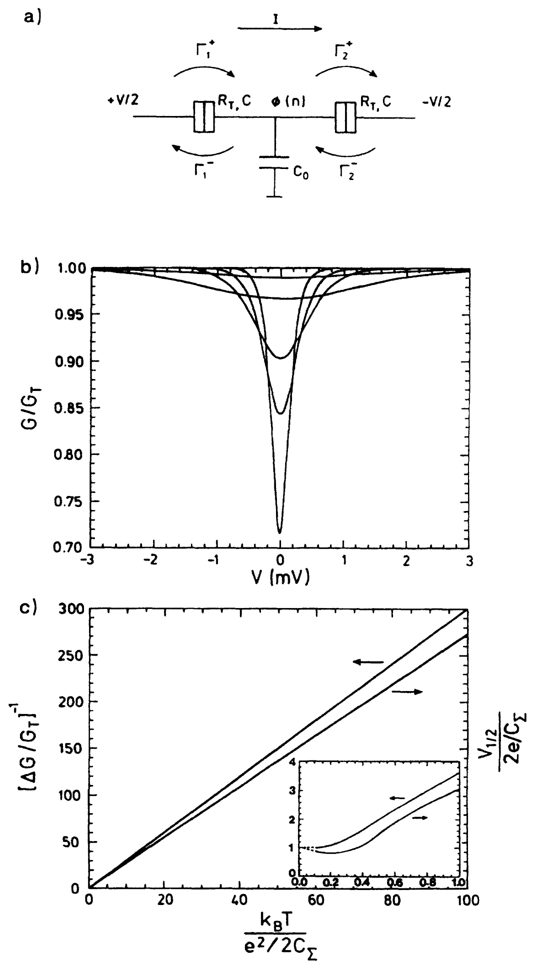


FIG. 1. (a) A symmetric double junction array. (b) Calculated conductance curves for two junctions with $C_\Sigma = 3$ fF at various temperatures. (c) The theoretical temperature dependencies of the relative change of conductance and the width of this feature of an arbitrary, symmetric two junction sample. The inset is a blowup of the low $T$ regime. For further details, see text.

to the first order, where the dimensionless parameter $v$ is defined as $v \equiv eV/2k_BT$, and $f(v) = [1 + \exp(v)(v - 1)]/[1 - \exp(v)]^2$. We obtain the $I$-$V$ curve by employing Eq. (4) and by noting that $\sum_{n=-\infty}^{\infty} \sigma(n)n = 0$, because $\sigma(-n) = \sigma(n)$, i.e.,

$$I = (k_BT/eR_T)\{v + u[f(v) - f(-v)]/2\} + \cdots . \quad (7)$$

Differentiating Eq. (7) we find the dependence of conductance $G/G_T$ at $v \neq 0$:

$$G/G_T = 1 - ug(v) + \cdots , \quad (8)$$

where

$$g(v) = -f'(v) = [v \sinh(v) - 4\sinh^2(v/2)]/8\sinh^4(v/2) \quad (9)$$

is symmetric in $v$ and $G_T \equiv (2R_T)^{-1}$. At $v = 0$ we obtain

$$G(v = 0)/G_T = 1 - u/6 + \cdots . \quad (10)$$

From Eqs. (8)–(10) we find $eV_{1/2}/2k_BT \simeq 5.44$, which gives $V_{1/2} \simeq 4.0$ mV at 4.21 K.

Figure 1(b) shows conductance calculated by the rigorous method for a fictitious double junction sample with $C_\Sigma = 3$ fF at five different temperatures of 10, 3, 1, 0.6, and 0.3 K. The results of the analytic expression are practically indistinguishable from these except at the two lowest temperatures where $u = 1.03$ and $u = 2.06$, respectively. In Fig. 1(c) we show the temperature dependencies in the general case for $V_{1/2}$, and for $(\Delta G/G_T)^{-1}(\equiv [1 - G(v = 0)/G_T]^{-1})$; serious deviations from the analytic linear dependences can be seen only at $u \gg 1$, as shown in the inset.

We do not present a rigorous derivation of the result for a long array. Yet, as a series connection of $N$ nonlinear resistances, it is obvious that $V_{1/2}$ is proportional to $N$ if all the tunnel resistances are equal [5]. This is supported by our Monte Carlo simulations as well. Experimentally a long array is more attractive because of the absence of higher order tunneling phenomena [2] and because larger $V_{1/2}$ is desirable at least toward lower temperatures.

In the experiments we have investigated several double and multijunction samples with capacitances ranging from $C_\Sigma = 0.3$ fF up to $C_\Sigma = 15$ fF. We have fabricated samples by electron beam lithography on various substrates making Al-oxide tunnel junctions between thin film Al conductors by the well known double angle evaporation techniques. The areas of the tunnel junctions were varied in the range $6 \times 10^{-3}$–$0.4\ \mu m^2$. At fixed points of temperature we employed superconducting transition temperatures at zero magnetic field of Pb at 7.19 K, Al at 1.18 K, and Ti at 0.39 K, and 4.21 K of boiling $^4$He at 760 mm Hg. The first three fixed points were detected by a mutual inductance bridge with an astatic pair of coils with one-half surrounding the samples. The conductance vs bias voltage was measured with a linear dc voltage sweep at typically ~5 min ramp time across the full bias range with a sufficiently low amplitude ac modulation ($V_{ac} \ll V_{1/2}$) typically at 30 Hz.

In Fig. 2 we see data on the experimental $N$ dependence of $V_{1/2}$ at $T = 4.2$ K for various samples with $\Delta G/G_T < 0.05$. The dashed line has a slope 2.03 mV/junction in fair agreement with our calculated value of $2 \times 1.98$ mV for two junctions. The measured value for two junction samples is offset by a few tenths of mV from the linear dependence. This probably originates from higher order tunneling processes, which rapidly become less significant with increasing $N$ and which were neglected in our analysis.

In Fig. 3 we show by circles the experimental temperature dependences of (a) the width $V_{1/2}$ and (b) the inverse height $(\Delta G/G_T)^{-1}$ of a typical sample with $N = 10$ at the four fixed temperatures, and the results of the rigorous symmetric calculation in the same temperature interval. The inset in (a) shows measured conductance curves around the four temperatures. $V_{1/2}$ of (a) does not involve

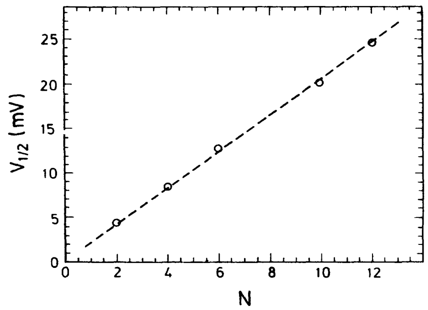


FIG. 2. Dependence of $V_{1/2}$ at $T = 4.21$ K on the number $N$ of junctions in the series array for samples where $\Delta G/G_T \ll 1$. The straight line through the measured points corresponds to $V_{1/2} = (4.06\ \mathrm{mV})N/2$.

any fit parameters making our thermometer a primary one, whereas in (b) the value of $\Delta G/G_T$ at one temperature, in this case at 4.21 K, gives $C_\Sigma \simeq 3.2$ fF whereby all the rest of the theoretical line is determined.

As to a thermometer, an important consideration is the tolerance of the results above to various nonidealities unavoidably present in the sensor. Experimentally, less than 5% variation of $V_{1/2}$ was observed from sample to sample at 4.2 K. Theoretically we can examine the effect of at least the following nonideality parameters: (1) asymmetry of the bias voltage, (2) deviation of the parameters of the two junctions $C_1 \neq C_2$, $R_{T1} \neq R_{T2}$, and (3) the background charge $Q_0$ of the island. In all these cases we employed a similar rigorous method as described above, requesting that $I_1$ and $I_2$ are equal.

Figure 4 illustrates the effects of nonidealities. An experimental $G/G_T$ curve of a 10 junction array at $T = 1.18$ K is shown by circles; $V_{1/2}$ is divided by 5 (= $N/2$). The solid line is the rigorous calculation for two junctions with $u = 0.435$, and the dashed line is the corresponding analytic expression. The two calculations agree mutually fairly well, within 4% in height and 3% in width, even with such a high value of $u$. Yet at the wings of the curve both calculations deviate somewhat from the measured curve. It may be caused by the asymmetry of the tunnel resistance; a ratio of $R_{T1}/R_{T2} = 1.7$ in the calculation gives next to a perfect fit to the measured curve. The other possibility is heating of the electrons by the power $P \simeq G_T V^2$, which may be significant at high bias voltages [6,7]. We have not yet investigated between the two alternatives. This deviation is not seen in all our samples.

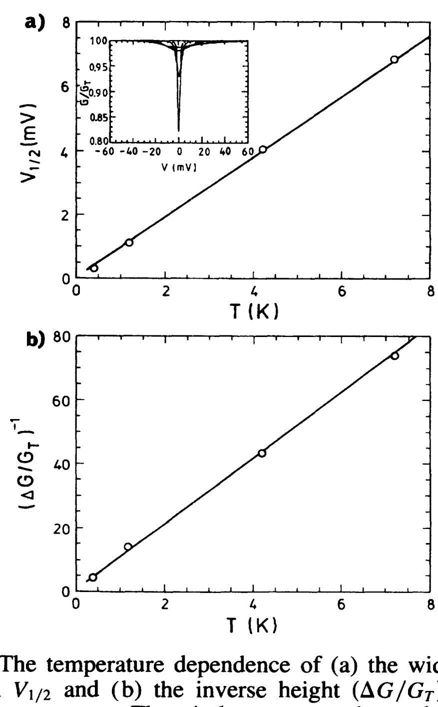


FIG. 3. The temperature dependence of (a) the width at half maximum $V_{1/2}$ and (b) the inverse height $(\Delta G/G_T)^{-1}$ of the conductance curves. The circles are experimental data of a sample with $N = 10$ junctions ($V_{1/2}$ is scaled by $N/2$), at the four fixed temperatures employed. The solid lines are results of our calculation with the fully symmetric double junction system. The inset in (a) shows measurements of $G/G_T$ around the four fixed temperatures.

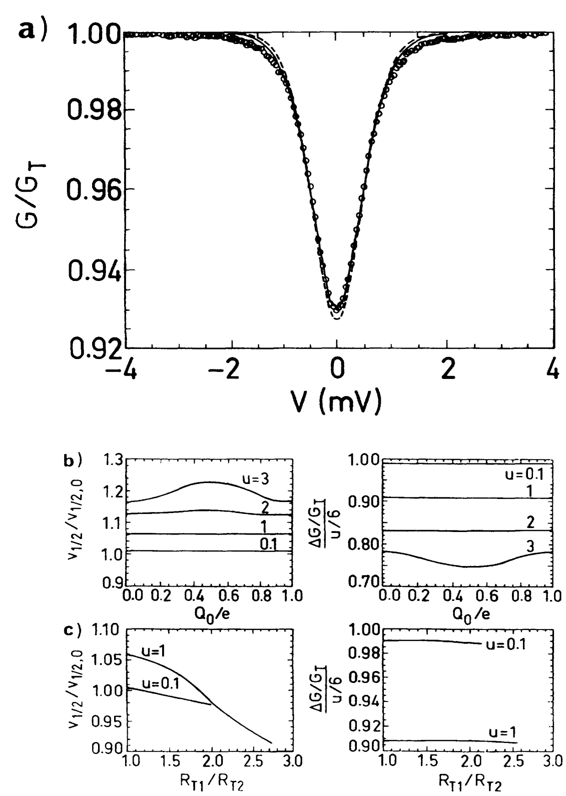


FIG. 4. (a) A conductance curve of a 10 junction sample at $T = 1.18$ K. Experimental data are displayed by circles. The solid line is the exact theoretical result for the fully symmetric case of Fig. 1 at $u = 0.435$, and the dashed line is the analytic result with the same value of parameter $u$. The effect of (b) the offset charge $Q_0$ and (c) the ratio of the two tunnel resistances $R_{T1}$ and $R_{T2}$, respectively, calculated with various values of $u$.

The background charge, $Q_0$, of the island is difficult to control experimentally, but it has negligible effect on the conductance curve when $u < 1$ as shown by our calculation in Fig. 4(b). Here we plot $v_{1/2}/v_{1/2,0}$, i.e., the width of the conductance minimum divided by the analytic width, and $(\Delta G/G_T)/(u/6)$, i.e., the relative height. Yet at low $T$ where $u \gg 1$, the $I$-$V$ curve is predominantly determined by $Q_0$. In this calculation, Eqs. (1)–(6) are modified such that $n$ is replaced by $n + Q_0/e$ on the right hand side of Eqs. (2) and (6).

The most serious of the nonidealities in the high $T$ regime seems to be the nonuniformity of the tunnel resistances. A few calculated results on this effect are shown in Fig. 4(c). We have checked experimentally the narrowing of the feature by an asymmetric layout of junction geometries. When the areas of the two junctions differed by a factor of 10, $V_{1/2}$ decreased by 15% at 4.2 K for a two junction sample with $u \simeq 0.2$.

On the premise of the theoretical results above, our thermometer is self-calibrating, and thus primary, at any temperature. There is only one temperature corresponding to any coordinate on the $(V_{1/2}, \Delta G/G_T)$ plane. $\Delta G/G_T$ can be used as a fast secondary thermometer calibrated at one temperature against $V_{1/2}$. The two quantities are fairly linear in $T$ or $T^{-1}$, respectively, over a wide temperature interval. The dynamic temperature range is determined on one hand by the signal to noise ratio to detect small changes of $G$ at the high temperature end, and, on the other hand, by the approach of Coulomb blockade at the low temperature end. By lock-in techniques we can measure conductance minima with 5% precision at $u \simeq 0.01$ ($\Delta G/G_T \simeq 0.2\%$) at high temperature end, and the Coulomb blockade limit is not yet approached when $u \simeq 3$. This gives the ratio of the maximum and the minimum measurable temperatures $T_{\max}/T_{\min} > 100$ by just one array. Yet we have another free parameter to choose, namely the mean of the temperature range, determined by the size of the junctions, i.e., by $C_\Sigma$.

An important question is the decoupling of the temperature of the electrons of the array from that of the substrate [6,7]. This problem is not quite as severe as in the case of very small tunnel junctions in general, because we can increase the island size toward lower temperatures.

Low temperature thermometers insensitive to magnetic field are fairly rare [8]; Coulomb blockade might provide one. We have tested one array at 4.2 K up to 8 T. The conductance curve was identical in all the cases to within our accuracy of 2% in width and 1% in height.

The international temperature scale ITS-90 is based on various fixed points down to 0.65 K. Primary thermometry at low temperatures, e.g., by nuclear ordering of $^{60}$Co or $^{54}$Mn, is often slow and difficult. Also, such methods as well as the various superconducting fixed points down to 16 mK are sensitive to even a modest magnetic field [8]. By our arrays measuring conductance provides simple primary thermometry in an adjustable temperature range on tiny on-chip sensors. We believe they will challenge the calibrated field independent (thin film) resistance or capacitance thermometers already commercially available.

Before our work, 2D arrays of superconducting tunnel junctions have been considered for thermometry by Delsing *et al.* [9], and the temperature dependence of a superconducting SET transistor was recently reported by Amar *et al.* [10].

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